

**Student  
Solutions Manual**  
to accompany  
**Applied Linear  
Statistical Models**  
Fifth Edition

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2005  
**McGraw-Hill/Irwin**  
Chicago, IL  
Boston, MA

# PREFACE

This Student Solutions Manual gives intermediate and final numerical results for all starred (\*) end-of-chapter Problems with computational elements contained in *Applied Linear Statistical Models*, 5th edition. No solutions are given for Exercises, Projects, or Case Studies.

In presenting calculational results we frequently show, for ease in checking, more digits than are significant for the original data. Students and other users may obtain slightly different answers than those presented here, because of different rounding procedures. When a problem requires a percentile (e.g. of the  $t$  or  $F$  distributions) not included in the Appendix B Tables, users may either interpolate in the table or employ an available computer program for finding the needed value. Again, slightly different values may be obtained than the ones shown here.

The data sets for all Problems, Exercises, Projects and Case Studies are contained in the compact disk provided with the text to facilitate data entry. It is expected that the student will use a computer or have access to computer output for all but the simplest data sets, where use of a basic calculator would be adequate. For most students, hands-on experience in obtaining the computations by computer will be an important part of the educational experience in the course.

While we have checked the solutions very carefully, it is possible that some errors are still present. We would be most grateful to have any errors called to our attention. Errata can be reported via the website for the book: <http://www.mhhe.com/KutnerALSM5e>.

We acknowledge with thanks the assistance of Lexin Li and Yingwen Dong in the checking of Chapters 1-14 of this manual. We, of course, are responsible for any errors or omissions that remain.

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# Chapter 1

## LINEAR REGRESSION WITH ONE PREDICTOR VARIABLE

- 1.20. a.  $\hat{Y} = -0.5802 + 15.0352X$   
d.  $\hat{Y}_h = 74.5958$
- 1.21. a.  $\hat{Y} = 10.20 + 4.00X$   
b.  $\hat{Y}_h = 14.2$   
c. 4.0  
d.  $(\bar{X}, \bar{Y}) = (1, 14.2)$
- 1.24. a. 

$i:$	1	2	...	44	45
$e_i:$	-9.4903	0.4392	...	1.4392	2.4039

  
 $\sum e_i^2 = 3416.377$   
 $\text{Min } Q = \sum e_i^2$   
b.  $MSE = 79.45063, \sqrt{MSE} = 8.913508, \text{ minutes}$
- 1.25. a.  $e_1 = 1.8000$   
b.  $\sum e_i^2 = 17.6000, MSE = 2.2000, \sigma^2$
- 1.27. a.  $\hat{Y} = 156.35 - 1.19X$   
b. (1)  $b_1 = -1.19$ , (2)  $\hat{Y}_h = 84.95$ , (3)  $e_8 = 4.4433$ ,  
(4)  $MSE = 66.8$



## Chapter 2

# INFERENCES IN REGRESSION AND CORRELATION ANALYSIS

- 2.5. a.  $t(.95; 43) = 1.6811$ ,  $15.0352 \pm 1.6811(.4831)$ ,  $14.2231 \leq \beta_1 \leq 15.8473$
- b.  $H_0: \beta_1 = 0$ ,  $H_a: \beta_1 \neq 0$ .  $t^* = (15.0352 - 0)/.4831 = 31.122$ . If  $|t^*| \leq 1.681$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+
- c. Yes
- d.  $H_0: \beta_1 \leq 14$ ,  $H_a: \beta_1 > 14$ .  $t^* = (15.0352 - 14)/.4831 = 2.1428$ . If  $t^* \leq 1.681$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = .0189
- 2.6. a.  $t(.975; 8) = 2.306$ ,  $b_1 = 4.0$ ,  $s\{b_1\} = .469$ ,  $4.0 \pm 2.306(.469)$ ,  
 $2.918 \leq \beta_1 \leq 5.082$
- b.  $H_0: \beta_1 = 0$ ,  $H_a: \beta_1 \neq 0$ .  $t^* = (4.0 - 0)/.469 = 8.529$ . If  $|t^*| \leq 2.306$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = .00003
- c.  $b_0 = 10.20$ ,  $s\{b_0\} = .663$ ,  $10.20 \pm 2.306(.663)$ ,  $8.671 \leq \beta_0 \leq 11.729$
- d.  $H_0: \beta_0 \leq 9$ ,  $H_a: \beta_0 > 9$ .  $t^* = (10.20 - 9)/.663 = 1.810$ . If  $t^* \leq 2.306$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .053
- e.  $H_0: \beta_1 = 0: \delta = |2 - 0|/.5 = 4$ , power = .93  
 $H_0: \beta_0 \leq 9: \delta = |11 - 9|/.75 = 2.67$ , power = .78
- 2.14. a.  $\hat{Y}_h = 89.6313$ ,  $s\{\hat{Y}_h\} = 1.3964$ ,  $t(.95; 43) = 1.6811$ ,  $89.6313 \pm 1.6811(1.3964)$ ,  
 $87.2838 \leq E\{Y_h\} \leq 91.9788$
- b.  $s\{\text{pred}\} = 9.0222$ ,  $89.6313 \pm 1.6811(9.0222)$ ,  $74.4641 \leq Y_{h(\text{new})} \leq 104.7985$ , yes,  
yes
- c.  $87.2838/6 = 14.5473$ ,  $91.9788/6 = 15.3298$ ,  $14.5473 \leq \text{Mean time per machine} \leq 15.3298$
- d.  $W^2 = 2F(.90; 2, 43) = 2(2.4304) = 4.8608$ ,  $W = 2.2047$ ,  $89.6313 \pm 2.2047(1.3964)$ ,  
 $86.5527 \leq \beta_0 + \beta_1 X_h \leq 92.7099$ , yes, yes
- 2.15. a.  $X_h = 2: \hat{Y}_h = 18.2$ ,  $s\{\hat{Y}_h\} = .663$ ,  $t(.995; 8) = 3.355$ ,  $18.2 \pm 3.355(.663)$ ,  $15.976 \leq E\{Y_h\} \leq 20.424$



- $X_h = 4$ :  $\hat{Y}_h = 26.2$ ,  $s\{\hat{Y}_h\} = 1.483$ ,  $26.2 \pm 3.355(1.483)$ ,  $21.225 \leq E\{Y_h\} \leq 31.175$
- b.  $s\{\text{pred}\} = 1.625$ ,  $18.2 \pm 3.355(1.625)$ ,  $12.748 \leq Y_{h(\text{new})} \leq 23.652$
- c.  $s\{\text{predmean}\} = 1.083$ ,  $18.2 \pm 3.355(1.083)$ ,  $14.567 \leq \bar{Y}_{h(\text{new})} \leq 21.833$ ,  $44 = 3(14.567) \leq \text{Total number of broken ampules} \leq 3(21.833) = 65$
- d.  $W^2 = 2F(.99; 2, 8) = 2(8.649) = 17.298$ ,  $W = 4.159$   
 $X_h = 2$ :  $18.2 \pm 4.159(.663)$ ,  $15.443 \leq \beta_0 + \beta_1 X_h \leq 20.957$   
 $X_h = 4$ :  $26.2 \pm 4.159(1.483)$ ,  $20.032 \leq \beta_0 + \beta_1 X_h \leq 32.368$   
 yes, yes

2.24. a.

Source	<i>SS</i>	<i>df</i>	<i>MS</i>
Regression	76,960.4	1	76,960.4
Error	3,416.38	43	79.4506
Total	80,376.78	44	

Source	<i>SS</i>	<i>df</i>	<i>MS</i>
Regression	76,960.4	1	76,960.4
Error	3,416.38	43	79.4506
Total	80,376.78	44	
Correction for mean	261,747.2	1	
Total, uncorrected	342,124	45	

- b.  $H_0: \beta_1 = 0$ ,  $H_a: \beta_1 \neq 0$ .  $F^* = 76,960.4/79.4506 = 968.66$ ,  $F(.90; 1, 43) = 2.826$ . If  $F^* \leq 2.826$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .
- c. 95.75% or 0.9575, coefficient of determination
- d. +.9785
- e.  $R^2$

2.25. a.

Source	<i>SS</i>	<i>df</i>	<i>MS</i>
Regression	160.00	1	160.00
Error	17.60	8	2.20
Total	177.60	9	

- b.  $H_0: \beta_1 = 0$ ,  $H_a: \beta_1 \neq 0$ .  $F^* = 160.00/2.20 = 72.727$ ,  $F(.95; 1, 8) = 5.32$ . If  $F^* \leq 5.32$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .
- c.  $t^* = (4.00 - 0)/.469 = 8.529$ ,  $(t^*)^2 = (8.529)^2 = 72.7 = F^*$
- d.  $R^2 = .9009$ ,  $r = .9492$ , 90.09%

2.27. a.

- $H_0: \beta_1 \geq 0$ ,  $H_a: \beta_1 < 0$ .  $s\{b_1\} = 0.090197$ ,  
 $t^* = (-1.19 - 0)/.090197 = -13.193$ ,  $t(.05; 58) = -1.67155$ .  
 If  $t^* \geq -1.67155$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  
 $P\text{-value} = 0+$

- c.  $t(.975; 58) = 2.00172$ ,  $-1.19 \pm 2.00172(.090197)$ ,  $-1.3705 \leq \beta_1 \leq -1.0095$

- 2.28. a.  $\hat{Y}_h = 84.9468$ ,  $s\{\hat{Y}_h\} = 1.05515$ ,  $t(.975; 58) = 2.00172$ ,  
 $84.9468 \pm 2.00172(1.05515)$ ,  $82.835 \leq E\{Y_h\} \leq 87.059$   
 b.  $s\{Y_{h(\text{new})}\} = 8.24101$ ,  $84.9468 \pm 2.00172(8.24101)$ ,  $68.451 \leq Y_{h(\text{new})} \leq 101.443$   
 c.  $W^2 = 2F(.95; 2, 58) = 2(3.15593) = 6.31186$ ,  $W = 2.512342$ ,  
 $84.9468 \pm 2.512342(1.05515)$ ,  $82.296 \leq \beta_0 + \beta_1 X_h \leq 87.598$ , yes, yes

2.29. a.

$i:$	1	2	...	59	60
$Y_i - \hat{Y}_i:$	0.823243	-1.55675	...	-0.666887	8.09309
$\hat{Y}_i - \bar{Y}:$	20.2101	22.5901	...	-14.2998	-19.0598

b.

Source	$SS$	$df$	$MS$
Regression	11,627.5	1	11,627.5
Error	3,874.45	58	66.8008
Total	15,501.95	59	

- c.  $H_0: \beta_1 = 0$ ,  $H_a: \beta_1 \neq 0$ .  $F^* = 11,627.5/66.8008 = 174.0623$ ,  
 $F(.90; 1, 58) = 2.79409$ . If  $F^* \leq 2.79409$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  
 d. 24.993% or .24993  
 e.  $R^2 = 0.750067$ ,  $r = -0.866064$

2.42. b. .95285,  $\rho_{12}$

- c.  $H_0: \rho_{12} = 0$ ,  $H_a: \rho_{12} \neq 0$ .  $t^* = (.95285\sqrt{13})/\sqrt{1 - (.95285)^2} = 11.32194$ ,  
 $t(.995; 13) = 3.012$ . If  $|t^*| \leq 3.012$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  
 d. No

2.44. a.  $H_0: \rho_{12} = 0$ ,  $H_a: \rho_{12} \neq 0$ .  $t^* = (.87\sqrt{101})/\sqrt{1 - (.87)^2} = 17.73321$ ,  $t(.95; 101) =$   
 1.663. If  $|t^*| \leq 1.663$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .

b.  $z' = 1.33308$ ,  $\sigma\{z'\} = .1$ ,  $z(.95) = 1.645$ ,  $1.33308 \pm 1.645(.1)$ ,  $1.16858 \leq \zeta \leq$   
 $1.49758$ ,  $.824 \leq \rho_{12} \leq .905$

c.  $.679 \leq \rho_{12}^2 \leq .819$

2.47. a. -0.866064,

b.  $H_0: \rho_{12} = 0$ ,  $H_a: \rho_{12} \neq 0$ .  $t^* = (-0.866064\sqrt{58})/\sqrt{1 - (-0.866064)^2} =$   
 $-13.19326$ ,  $t(.975; 58) = 2.00172$ . If  $|t^*| \leq 2.00172$  conclude  $H_0$ , otherwise  $H_a$ .  
 Conclude  $H_a$ .

c. -0.8657217

d.  $H_0$ : There is no association between  $X$  and  $Y$

$H_a$ : There is an association between  $X$  and  $Y$

$t^* = \frac{-0.8657217\sqrt{58}}{\sqrt{1 - (-0.8657217)^2}} = -13.17243$ .  $t(0.975, 58) = 2.001717$ . If  $|t^*| \leq$   
 $2.001717$ , conclude  $H_0$ , otherwise, conclude  $H_a$ . Conclude  $H_a$ .



# Chapter 3

## DIAGNOSTICS AND REMEDIAL MEASURES

3.4.c and d.

$i:$	1	2	...	44	45
$\hat{Y}_i:$	29.49034	59.56084	...	59.56084	74.59608
$e_i:$	-9.49034	0.43916	...	1.43916	2.40392

e.

Ascending order:	1	2	...	44	45
Ordered residual:	-22.77232	-19.70183	...	14.40392	15.40392
Expected value:	-19.63272	-16.04643	...	16.04643	19.63272

$H_0$ : Normal,  $H_a$ : not normal.  $r = 0.9891$ . If  $r \geq .9785$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

g.  $SSR^* = 15,155$ ,  $SSE = 3416.38$ ,  $X_{BP}^2 = (15,155/2) \div (3416.38/45)^2 = 1.314676$ ,  $\chi^2(.95; 1) = 3.84$ . If  $X_{BP}^2 \leq 3.84$  conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

3.5. c.

$i:$	1	2	3	4	5	6	7	8	9	10
$e_i:$	1.8	-1.2	-1.2	1.8	-2	-1.2	-2.2	.8	.8	.8

e.

Ascending Order:	1	2	3	4	5	6	7	8	9	10
Ordered residual:	-2.2	-1.2	-1.2	-1.2	-2	.8	.8	.8	1.8	1.8
Expected value:	-2.3	-1.5	-1.0	-.6	-.2	.2	.6	1.0	1.5	2.3

$H_0$ : Normal,  $H_a$ : not normal.  $r = .961$ . If  $r \geq .879$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

g.  $SSR^* = 6.4$ ,  $SSE = 17.6$ ,  $X_{BP}^2 = (6.4/2) \div (17.6/10)^2 = 1.03$ ,  $\chi^2(.90; 1) = 2.71$ . If  $X_{BP}^2 \leq 2.71$  conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

Yes.

3.7.b and c.

$i:$	1	2	...	59	60
$e_i:$	0.82324	-1.55675	...	-0.66689	8.09309
$\hat{Y}_i:$	105.17676	107.55675	...	70.66689	65.90691

d.

Ascending order:	1	2	...	59	60
Ordered residual:	-16.13683	-13.80686	...	13.95312	23.47309
Expected value:	-18.90095	-15.75218	...	15.75218	18.90095

$H_0$ : Normal,  $H_a$ : not normal.  $r = 0.9897$ . If  $r \geq 0.984$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

e.  $SSR^* = 31,833.4$ ,  $SSE = 3,874.45$ ,

$X_{BP}^2 = (31,833.4/2) \div (3,874.45/60)^2 = 3.817116$ ,  $\chi^2(.99; 1) = 6.63$ . If  $X_{BP}^2 \leq 6.63$  conclude error variance constant, otherwise error variance not constant. Conclude error variance constant. Yes.

3.13. a.  $H_0: E\{Y\} = \beta_0 + \beta_1X$ ,  $H_a: E\{Y\} \neq \beta_0 + \beta_1X$

b.  $SSPE = 2797.66$ ,  $SSLF = 618.719$ ,  $F^* = (618.719/8) \div (2797.66/35) = 0.967557$ ,  $F(.95; 8, 35) = 2.21668$ . If  $F^* \leq 2.21668$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

3.17. b.

$\lambda:$	.3	.4	.5	.6	.7
$SSE:$	1099.7	967.9	916.4	942.4	1044.2

c.  $\hat{Y}' = 10.26093 + 1.07629X$

e.

$i:$	1	2	3	4	5
$e_i:$	-.36	.28	.31	-.15	.30
$\hat{Y}'_i:$	10.26	11.34	12.41	13.49	14.57
Expected value:	-.24	.14	.36	-.14	.24
$i:$	6	7	8	9	10
$e_i:$	-.41	.10	-.47	.47	-.07
$\hat{Y}'_i:$	15.64	16.72	17.79	18.87	19.95
Expected value:	-.36	.04	-.56	.56	-.04

f.  $\hat{Y} = (10.26093 + 1.07629X)^2$

# Chapter 4

## SIMULTANEOUS INFERENCES AND OTHER TOPICS IN REGRESSION ANALYSIS

- 4.3. a. Opposite directions, negative tilt  
b.  $B = t(.9875; 43) = 2.32262$ ,  $b_0 = -0.580157$ ,  $s\{b_0\} = 2.80394$ ,  $b_1 = 15.0352$ ,  $s\{b_1\} = 0.483087$   
 $-0.580157 \pm 2.32262(2.80394)$        $-7.093 \leq \beta_0 \leq 5.932$   
 $15.0352 \pm 2.32262(0.483087)$        $13.913 \leq \beta_1 \leq 16.157$   
c. Yes
- 4.4. a. Opposite directions, negative tilt  
b.  $B = t(.9975; 8) = 3.833$ ,  $b_0 = 10.2000$ ,  $s\{b_0\} = .6633$ ,  $b_1 = 4.0000$ ,  $s\{b_1\} = .4690$   
 $10.2000 \pm 3.833(.6633)$        $7.658 \leq \beta_0 \leq 12.742$   
 $4.0000 \pm 3.833(.4690)$        $2.202 \leq \beta_1 \leq 5.798$
- 4.6. a.  $B = t(.9975; 14) = 2.91839$ ,  $b_0 = 156.347$ ,  $s\{b_0\} = 5.51226$ ,  $b_1 = -1.190$ ,  $s\{b_1\} = 0.0901973$   
 $156.347 \pm 2.91839(5.51226)$        $140.260 \leq \beta_0 \leq 172.434$   
 $-1.190 \pm 2.91839(0.0901973)$        $-1.453 \leq \beta_1 \leq -0.927$   
b. Opposite directions  
c. No
- 4.7. a.  $F(.90; 2, 43) = 2.43041$ ,  $W = 2.204727$   
 $X_h = 3$ :  $44.5256 \pm 2.204727(1.67501)$        $40.833 \leq E\{Y_h\} \leq 48.219$   
 $X_h = 5$ :  $74.5961 \pm 2.204727(1.32983)$        $71.664 \leq E\{Y_h\} \leq 77.528$   
 $X_h = 7$ :  $104.667 \pm 2.204727(1.6119)$        $101.113 \leq E\{Y_h\} \leq 108.221$   
b.  $F(.90; 2, 43) = 2.43041$ ,  $S = 2.204727$ ;  $B = t(.975; 43) = 2.01669$ ; Bonferroni  
c.  $X_h = 4$ :  $59.5608 \pm 2.01669(9.02797)$        $41.354 \leq Y_{h(\text{new})} \leq 77.767$

$$X_h = 7: 104.667 \pm 2.01669(9.05808) \quad 86.3997 \leq Y_{h(\text{new})} \leq 122.934$$

4.8. a.  $F(.95; 2, 8) = 4.46, W = 2.987$

$$X_h = 0: 10.2000 \pm 2.987(.6633) \quad 8.219 \leq E\{Y_h\} \leq 12.181$$

$$X_h = 1: 14.2000 \pm 2.987(.4690) \quad 12.799 \leq E\{Y_h\} \leq 15.601$$

$$X_h = 2: 18.2000 \pm 2.987(.6633) \quad 16.219 \leq E\{Y_h\} \leq 20.181$$

b.  $B = t(.99167; 8) = 3.016, \text{ yes}$

c.  $F(.95; 3, 8) = 4.07, S = 3.494$

$$X_h = 0: 10.2000 \pm 3.494(1.6248) \quad 4.523 \leq Y_{h(\text{new})} \leq 15.877$$

$$X_h = 1: 14.2000 \pm 3.494(1.5556) \quad 8.765 \leq Y_{h(\text{new})} \leq 19.635$$

$$X_h = 2: 18.2000 \pm 3.494(1.6248) \quad 12.523 \leq Y_{h(\text{new})} \leq 23.877$$

d.  $B = 3.016, \text{ yes}$

4.10. a.  $F(.95; 2, 58) = 3.15593, W = 2.512342$

$$X_h = 45: 102.797 \pm 2.512342(1.71458) \quad 98.489 \leq E\{Y_h\} \leq 107.105$$

$$X_h = 55: 90.8968 \pm 2.512342(1.1469) \quad 88.015 \leq E\{Y_h\} \leq 93.778$$

$$X_h = 65: 78.9969 \pm 2.512342(1.14808) \quad 76.113 \leq E\{Y_h\} \leq 81.881$$

b.  $B = t(.99167; 58) = 2.46556, \text{ no}$

c.  $B = 2.46556$

$$X_h = 48: 99.2268 \pm 2.46556(8.31158) \quad 78.734 \leq Y_{h(\text{new})} \leq 119.720$$

$$X_h = 59: 86.1368 \pm 2.46556(8.24148) \quad 65.817 \leq Y_{h(\text{new})} \leq 106.457$$

$$X_h = 74: 68.2869 \pm 2.46556(8.33742) \quad 47.730 \leq Y_{h(\text{new})} \leq 88.843$$

d. Yes, yes

4.16. a.  $\hat{Y} = 14.9472X$

b.  $s\{b_1\} = 0.226424, t(.95; 44) = 1.68023, 14.9472 \pm 1.68023(0.226424), 14.567 \leq \beta_1 \leq 15.328$

c.  $\hat{Y}_h = 89.6834, s\{\text{pred}\} = 8.92008, 89.6834 \pm 1.68023(8.92008), 74.696 \leq Y_{h(\text{new})} \leq 104.671$

4.17. b.

$i:$	1	2	...	44	45
$e_i:$	-9.89445	0.21108	...	1.2111	2.2639

No

c.  $H_0: E\{Y\} = \beta_1 X, H_a: E\{Y\} \neq \beta_1 X. SSLF = 622.12, SSPE = 2797.66, F^* = (622.12/9) \div (2797.66/35) = 0.8647783, F(.99; 9, 35) = 2.96301. \text{ If } F^* \leq 2.96301 \text{ conclude } H_0, \text{ otherwise } H_a. \text{ Conclude } H_0. P\text{-value} = 0.564$

# Chapter 5

## MATRIX APPROACH TO SIMPLE LINEAR REGRESSION ANALYSIS

$$5.4. \quad (1) 503.77 \quad (2) \begin{bmatrix} 5 & 0 \\ 0 & 160 \end{bmatrix} \quad (3) \begin{bmatrix} 49.7 \\ -39.2 \end{bmatrix}$$

$$5.6. \quad (1) 2,194 \quad (2) \begin{bmatrix} 10 & 10 \\ 10 & 20 \end{bmatrix} \quad (3) \begin{bmatrix} 142 \\ 182 \end{bmatrix}$$

$$5.12. \quad \begin{bmatrix} .2 & 0 \\ 0 & .00625 \end{bmatrix}$$

$$5.14. \quad \text{a.} \quad \begin{bmatrix} 4 & 7 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 25 \\ 12 \end{bmatrix}$$

$$\text{b.} \quad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 4.5 \\ 1 \end{bmatrix}$$

$$5.18. \quad \text{a.} \quad \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix}$$

$$\text{b.} \quad \mathbf{E} \left\{ \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} \right\} = \begin{bmatrix} \frac{1}{4}[E\{Y_1\} + E\{Y_2\} + E\{Y_3\} + E\{Y_4\}] \\ \frac{1}{2}[E\{Y_1\} + E\{Y_2\} - E\{Y_3\} - E\{Y_4\}] \end{bmatrix}$$

$$\text{c.} \quad \sigma^2\{\mathbf{W}\} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} \sigma^2\{Y_1\} & \sigma\{Y_1, Y_2\} & \sigma\{Y_1, Y_3\} & \sigma\{Y_1, Y_4\} \\ \sigma\{Y_2, Y_1\} & \sigma^2\{Y_2\} & \sigma\{Y_2, Y_3\} & \sigma\{Y_2, Y_4\} \\ \sigma\{Y_3, Y_1\} & \sigma\{Y_3, Y_2\} & \sigma^2\{Y_3\} & \sigma\{Y_3, Y_4\} \\ \sigma\{Y_4, Y_1\} & \sigma\{Y_4, Y_2\} & \sigma\{Y_4, Y_3\} & \sigma^2\{Y_4\} \end{bmatrix} \\ \times \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} \end{bmatrix}$$

Using the notation  $\sigma_1^2$  for  $\sigma^2\{Y_1\}$ ,  $\sigma_{12}$  for  $\sigma\{Y_1, Y_2\}$ , etc., we obtain:

$$\sigma^2\{W_1\} = \frac{1}{16}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 + 2\sigma_{12} + 2\sigma_{13} + 2\sigma_{14} + 2\sigma_{23} + 2\sigma_{24} + 2\sigma_{34})$$



$$\sigma^2\{W_2\} = \frac{1}{4}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 + 2\sigma_{12} - 2\sigma_{13} - 2\sigma_{14} - 2\sigma_{23} - 2\sigma_{24} + 2\sigma_{34})$$

$$\sigma\{W_1, W_2\} = \frac{1}{8}(\sigma_1^2 + \sigma_2^2 - \sigma_3^2 - \sigma_4^2 + 2\sigma_{12} - 2\sigma_{34})$$

5.19.  $\begin{bmatrix} 3 & 5 \\ 5 & 17 \end{bmatrix}$

5.21.  $5Y_1^2 + 4Y_1Y_2 + Y_2^2$

5.23. a. (1)  $\begin{bmatrix} 9.940 \\ -2.245 \end{bmatrix}$  (2)  $\begin{bmatrix} -.18 \\ .04 \\ .26 \\ .08 \\ -.20 \end{bmatrix}$  (3) 9.604 (4) .148

(5)  $\begin{bmatrix} .00987 & 0 \\ 0 & .000308 \end{bmatrix}$  (6) 11.41 (7) .02097

c.  $\begin{bmatrix} .6 & .4 & .2 & 0 & -.2 \\ .4 & .3 & .2 & .1 & 0 \\ .2 & .2 & .2 & .2 & .2 \\ 0 & .1 & .2 & .3 & .4 \\ -.2 & 0 & .2 & .4 & .6 \end{bmatrix}$

d.  $\begin{bmatrix} .01973 & -.01973 & -.00987 & .00000 & .00987 \\ -.01973 & .03453 & -.00987 & -.00493 & .00000 \\ -.00987 & -.00987 & .03947 & -.00987 & -.00987 \\ .00000 & -.00493 & -.00987 & .03453 & -.01973 \\ .00987 & .00000 & -.00987 & -.01973 & .01973 \end{bmatrix}$

5.25. a. (1)  $\begin{bmatrix} .2 & -.1 \\ -.1 & .1 \end{bmatrix}$  (2)  $\begin{bmatrix} 10.2 \\ 4.0 \end{bmatrix}$  (3)  $\begin{bmatrix} 1.8 \\ -1.2 \\ -1.2 \\ 1.8 \\ -.2 \\ -1.2 \\ -2.2 \\ .8 \\ .8 \\ .8 \end{bmatrix}$

(4)  $\begin{bmatrix} .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 \\ .1 & .2 & 0 & .2 & -.1 & .1 & .2 & .1 & 0 & .2 \\ .1 & 0 & .2 & 0 & .3 & .1 & 0 & .1 & .2 & 0 \\ .1 & .2 & 0 & .2 & -.1 & .1 & .2 & .1 & 0 & .2 \\ .1 & -.1 & .3 & -.1 & .5 & .1 & -.1 & .1 & .3 & -.1 \\ .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 \\ .1 & .2 & 0 & .2 & -.1 & .1 & .2 & .1 & 0 & .2 \\ .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 \\ .1 & .0 & .2 & 0 & .3 & .1 & 0 & .1 & .2 & 0 \\ .1 & .2 & 0 & .2 & -.1 & .1 & .2 & .1 & .0 & .2 \end{bmatrix}$

$$(5) 17.60 \quad (6) \begin{bmatrix} .44 & -.22 \\ -.22 & .22 \end{bmatrix} \quad (7) 18.2 \quad (8) .44$$

$$\text{b. } (1) .22 \quad (2) -.22 \quad (3) .663$$

$$\text{c. } \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .1 & -.1 & .1 & -.2 & 0 & .1 & 0 & -.1 & .1 \\ 0 & -.1 & .1 & -.1 & .2 & 0 & -.1 & 0 & .1 & -.1 \\ 0 & .1 & -.1 & .1 & -.2 & 0 & .1 & 0 & -.1 & .1 \\ 0 & -.2 & .2 & -.2 & .4 & 0 & -.2 & 0 & .2 & -.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .1 & -.1 & .1 & -.2 & 0 & .1 & 0 & -.1 & .1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -.1 & .1 & -.1 & .2 & 0 & -.1 & 0 & .1 & -.1 \\ 0 & .1 & -.1 & .1 & -.2 & 0 & .1 & 0 & -.1 & .1 \end{bmatrix}$$



# Chapter 6

## MULTIPLE REGRESSION – I

6.9. c. 
$$\begin{matrix} Y \\ X_1 \\ X_2 \\ X_3 \end{matrix} \begin{bmatrix} 1.0000 & .2077 & .0600 & .8106 \\ & 1.0000 & .0849 & .0457 \\ & & 1.0000 & .1134 \\ & & & 1.0000 \end{bmatrix}$$

6.10. a.  $\hat{Y} = 4149.89 + 0.000787X_1 - 13.166X_2 + 623.554X_3$

b&c.

$i:$	1	2	...	51	52
$e_i:$	-32.0635	169.2051	...	-184.8776	64.5168
Expected Val.:	-24.1737	151.0325	...	-212.1315	75.5358

e.  $n_1 = 26, \bar{d}_1 = 145.0, n_2 = 26, \bar{d}_2 = 77.4, s = 81.7,$   
 $t_{BF}^* = (145.0 - 77.4)/[81.7\sqrt{(1/26) + (1/26)}] = 2.99, t(.995; 50) = 2.67779.$  If  $|t_{BF}^*| \leq 2.67779$  conclude error variance constant, otherwise error variance not constant. Conclude error variance not constant.

6.11. a.  $H_0: \beta_1 = \beta_2 = \beta_3 = 0, H_a:$  not all  $\beta_k = 0$  ( $k = 1, 2, 3$ ).  $MSR = 725, 535,$   
 $MSE = 20, 531.9, F^* = 725, 535/20, 531.9 = 35.337, F(.95; 3, 48) = 2.79806.$  If  $F^* \leq 2.79806$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+.

b.  $s\{b_1\} = .000365, s\{b_3\} = 62.6409, B = t(.9875; 48) = 2.3139$

$$0.000787 \pm 2.3139(.000365) \quad - .000058 \leq \beta_1 \leq 0.00163$$

$$623.554 \pm 2.3139(62.6409) \quad 478.6092 \leq \beta_3 \leq 768.4988$$

c.  $SSR = 2, 176, 606, SSTO = 3, 162, 136, R^2 = .6883$

6.12. a.  $F(.95; 4, 48) = 2.56524, W = 3.2033; B = t(.995; 48) = 2.6822$

$X_{h1}$	$X_{h2}$	$X_{h3}$	
302,000	7.2	0:	$4292.79 \pm 2.6822(21.3567) \quad 4235.507 \leq E\{Y_h\} \leq 4350.073$
245,000	7.4	0:	$4245.29 \pm 2.6822(29.7021) \quad 4165.623 \leq E\{Y_h\} \leq 4324.957$
280,000	6.9	0:	$4279.42 \pm 2.6822(24.4444) \quad 4213.855 \leq E\{Y_h\} \leq 4344.985$
350,000	7.0	0:	$4333.20 \pm 2.6822(28.9293) \quad 4255.606 \leq E\{Y_h\} \leq 4410.794$
295,000	6.7	1:	$4917.42 \pm 2.6822(62.4998) \quad 4749.783 \leq E\{Y_h\} \leq 5085.057$

b. Yes, no

6.13.  $F(.95; 4, 48) = 2.5652, S = 3.2033; B = t(.99375; 48) = 2.5953$

$X_{h1}$	$X_{h2}$	$X_{h3}$		
230,000	7.5	0:	$4232.17 \pm 2.5953(147.288)$	$3849.913 \leq Y_{h(\text{new})} \leq 4614.427$
250,000	7.3	0:	$4250.55 \pm 2.5953(146.058)$	$3871.486 \leq Y_{h(\text{new})} \leq 4629.614$
280,000	7.1	0:	$4276.79 \pm 2.5953(145.134)$	$3900.124 \leq Y_{h(\text{new})} \leq 4653.456$
340,000	6.9	0:	$4326.65 \pm 2.5953(145.930)$	$3947.918 \leq Y_{h(\text{new})} \leq 4705.382$

6.14. a.  $\hat{Y}_h = 4278.37, s\{\text{predmean}\} = 85.82262, t(.975; 48) = 2.01063,$

$4278.37 \pm 2.01063(85.82262), 4105.812 \leq \bar{Y}_{h(\text{new})} \leq 4450.928$

b.  $12317.44 \leq \text{Total labor hours} \leq 13352.78$

6.15. b. 
$$Y \begin{bmatrix} 1.000 & -.7868 & -.6029 & -.6446 \\ X_1 & & 1.000 & .5680 & .5697 \\ X_2 & & & 1.000 & .6705 \\ X_3 & & & & 1.000 \end{bmatrix}$$

c.  $\hat{Y} = 158.491 - 1.1416X_1 - 0.4420X_2 - 13.4702X_3$

d&e.

$i:$	1	2	...	45	46
$e_i:$	.1129	-9.0797	...	-5.5380	10.0524
Expected Val.:	-0.8186	-8.1772	...	-5.4314	8.1772

f. No

g.  $SSR^* = 21,355.5, SSE = 4,248.8, X_{BP}^2 = (21,355.5/2) \div (4,248.8 / 46)^2 = 1.2516, \chi^2(.99; 3) = 11.3449.$  If  $X_{BP}^2 \leq 11.3449$  conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

6.16. a.  $H_0: \beta_1 = \beta_2 = \beta_3 = 0, H_a: \text{not all } \beta_k = 0 (k = 1, 2, 3).$

$MSR = 3,040.2, MSE = 101.2, F^* = 3,040.2/101.2 = 30.05, F(.90; 3, 42) = 2.2191.$  If  $F^* \leq 2.2191$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0.4878

b.  $s\{b_1\} = .2148, s\{b_2\} = .4920, s\{b_3\} = 7.0997, B = t(.9833; 42) = 2.1995$

$-1.1416 \pm 2.1995(.2148) \quad -1.6141 \leq \beta_1 \leq -0.6691$

$-.4420 \pm 2.1995(.4920) \quad -1.5242 \leq \beta_2 \leq 0.6402$

$-13.4702 \pm 2.1995(7.0997) \quad -29.0860 \leq \beta_3 \leq 2.1456$

c.  $SSR = 9,120.46, SSTO = 13,369.3, R = .8260$

6.17. a.  $\hat{Y}_h = 69.0103, s\{\hat{Y}_h\} = 2.6646, t(.95; 42) = 1.6820, 69.0103 \pm 1.6820(2.6646), 64.5284 \leq E\{Y_h\} \leq 73.4922$

b.  $s\{\text{pred}\} = 10.405, 69.0103 \pm 1.6820(10.405), 51.5091 \leq Y_{h(\text{new})} \leq 86.5115$

# Chapter 7

## MULTIPLE REGRESSION – II

- 7.4. a.  $SSR(X_1) = 136,366$ ,  $SSR(X_3|X_1) = 2,033,566$ ,  $SSR(X_2|X_1, X_3) = 6,674$ ,  
 $SSE(X_1, X_2, X_3) = 985,530$ ,  $df: 1, 1, 1, 48$ .
- b.  $H_0: \beta_2 = 0$ ,  $H_a: \beta_2 \neq 0$ .  $SSR(X_2|X_1, X_3) = 6,674$ ,  $SSE(X_1, X_2, X_3) = 985,530$ ,  
 $F^* = (6,674/1) \div (985,530/48) = 0.32491$ ,  $F(.95; 1, 17) = 4.04265$ . If  $F^* \leq 4.04265$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = 0.5713.
- c. Yes,  $SSR(X_1) + SSR(X_2|X_1) = 136,366 + 5,726 = 142,092$ ,  $SSR(X_2) + SSR(X_1|X_2) = 11,394.9 + 130,697.1 = 142,092$ .  
Yes.
- 7.5. a.  $SSR(X_2) = 4,860.26$ ,  $SSR(X_1|X_2) = 3,896.04$ ,  $SSR(X_3|X_2, X_1) = 364.16$ ,  
 $SSE(X_1, X_2, X_3) = 4,248.84$ ,  $df: 1, 1, 1, 42$
- b.  $H_0: \beta_3 = 0$ ,  $H_a: \beta_3 \neq 0$ .  $SSR(X_3|X_1, X_2) = 364.16$ ,  $SSE(X_1, X_2, X_3) = 4,248.84$ ,  
 $F^* = (364.16/1) \div (4,248.84/42) = 3.5997$ ,  $F(.975; 1, 42) = 5.4039$ . If  $F^* \leq 5.4039$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = 0.065.
- 7.6.  $H_0: \beta_2 = \beta_3 = 0$ ,  $H_a: \text{not both } \beta_2 \text{ and } \beta_3 = 0$ .  $SSR(X_2, X_3|X_1) = 845.07$ ,  
 $SSE(X_1, X_2, X_3) = 4,248.84$ ,  $F^* = (845.07/2) \div (4,248.84/42) = 4.1768$ ,  $F(.975; 2, 42) = 4.0327$ . If  $F^* \leq 4.0327$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0.022.
- 7.9.  $H_0: \beta_1 = -1.0$ ,  $\beta_2 = 0$ ;  $H_a: \text{not both equalities hold}$ . Full model:  $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \varepsilon_i$ , reduced model:  $Y_i + X_{i1} = \beta_0 + \beta_3 X_{i3} + \varepsilon_i$ .  $SSE(F) = 4,248.84$ ,  
 $df_F = 42$ ,  $SSE(R) = 4,427.7$ ,  $df_R = 44$ ,  $F^* = [(4427.7 - 4248.84)/2] \div (4,248.84/42) = .8840$ ,  $F(.975; 2, 42) = 4.0327$ . If  $F^* \leq 4.0327$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .
- 7.13.  $R_{Y_1}^2 = .0431$ ,  $R_{Y_2}^2 = .0036$ ,  $R_{12}^2 = .0072$ ,  $R_{Y_1|2}^2 = 0.0415$ ,  $R_{Y_2|1}^2 = 0.0019$ ,  $R_{Y_2|13}^2 = .0067$   $R^2 = .6883$
- 7.14. a.  $R_{Y_1}^2 = .6190$ ,  $R_{Y_1|2}^2 = .4579$ ,  $R_{Y_1|23}^2 = .4021$   
b.  $R_{Y_2}^2 = .3635$ ,  $R_{Y_2|1}^2 = .0944$ ,  $R_{Y_2|13}^2 = .0189$
- 7.17. a.  $\hat{Y}^* = .17472X_1^* - .04639X_2^* + .80786X_3^*$   
b.  $R_{12}^2 = .0072$ ,  $R_{13}^2 = .0021$ ,  $R_{23}^2 = .0129$

- c.  $s_Y = 249.003$ ,  $s_1 = 55274.6$ ,  $s_2 = .87738$ ,  $s_3 = .32260$   $b_1 = \frac{249.003}{55274.6}(.17472) = .00079$ ,  $b_2 = \frac{249.003}{.87738}(-.04639) = -13.16562$ ,  $b_3 = \frac{249.003}{.32260}(.80786) = 623.5572$ ,  
 $b_0 = 4363.04 - .00079(302, 693) + 13.16562(7.37058) - 623.5572(0.115385) = 4149.002$ .
- 7.18. a.  $\hat{Y}^* = -.59067X_1^* - .11062X_2^* - .23393X_3^*$   
 b.  $R_{12}^2 = .32262$ ,  $R_{13}^2 = .32456$ ,  $R_{23}^2 = .44957$   
 c.  $s_Y = 17.2365$ ,  $s_1 = 8.91809$ ,  $s_2 = 4.31356$ ,  $s_3 = .29934$ ,  $b_1 = \frac{17.2365}{8.91809}(-.59067) = -1.14162$ ,  $b_2 = \frac{17.2365}{4.31356}(-.11062) = -.44203$ ,  $b_3 = \frac{17.2365}{.29934}(-.23393) = -13.47008$ ,  
 $b_0 = 61.5652 + 1.14162(38.3913) + .44203(50.4348) + 13.47008(2.28696) = 158.4927$
- 7.25. a.  $\hat{Y} = 4079.87 + 0.000935X_2$   
 c. No,  $SSR(X_1) = 136, 366$ ,  $SSR(X_1|X_2) = 130, 697$   
 d.  $r_{12} = .0849$
- 7.26. a.  $\hat{Y} = 156.672 - 1.26765X_1 - 0.920788X_2$   
 c. No,  $SSR(X_1) = 8, 275.3$ ,  $SSR(X_1|X_3) = 3, 483.89$   
 No,  $SSR(X_2) = 4, 860.26$ ,  $SSR(X_2|X_3) = 708$   
 d.  $r_{12} = .5680$ ,  $r_{13} = .5697$ ,  $r_{23} = .6705$

# Chapter 8

## MODELS FOR QUANTITATIVE AND QUALITATIVE PREDICTORS

- 8.4. a.  $\hat{Y} = 82.9357 - 1.18396x + .0148405x^2$ ,  $R^2 = .76317$
- b.  $H_0: \beta_1 = \beta_{11} = 0$ ,  $H_a$ : not both  $\beta_1$  and  $\beta_{11} = 0$ .  $MSR = 5915.31$ ,  $MSE = 64.409$ ,  $F^* = 5915.31/64.409 = 91.8398$ ,  $F(.95; 2, 57) = 3.15884$ . If  $F^* \leq 3.15884$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .
- c.  $\hat{Y}_h = 99.2546$ ,  $s\{\hat{Y}_h\} = 1.4833$ ,  $t(.975; 57) = 2.00247$ ,  $99.2546 \pm 2.00247(1.4833)$ ,  $96.2843 \leq E\{Y_h\} \leq 102.2249$
- d.  $s\{\text{pred}\} = 8.16144$ ,  $99.2546 \pm 2.00247(8.16144)$ ,  $82.91156 \leq Y_{h(\text{new})} \leq 115.5976$
- e.  $H_0: \beta_{11} = 0$ ,  $H_a: \beta_{11} \neq 0$ .  $s\{b_{11}\} = .00836$ ,  $t^* = .0148405/.00836 = 1.7759$ ,  $t(.975; 57) = 2.00247$ . If  $|t^*| \leq 2.00247$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ . Alternatively,  $SSR(x^2|x) = 203.1$ ,  $SSE(x, x^2) = 3671.31$ ,  $F^* = (203.1/1) \div (3671.31/57) = 3.15329$ ,  $F(.95; 1, 57) = 4.00987$ . If  $F^* \leq 4.00987$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .
- f.  $\hat{Y} = 207.350 - 2.96432X + .0148405X^2$
- g.  $r_{X, X^2} = .9961$ ,  $r_{x, x^2} = -.0384$
- 8.5. a. 

$i:$	1	2	3	...	58	59	60
$e_i:$	-1.3238	-4.7592	-3.8091	...	-11.7798	-.8515	6.22023
- b.  $H_0: E\{Y\} = \beta_0 + \beta_1x + \beta_{11}x^2$ ,  $H_a: E\{Y\} \neq \beta_0 + \beta_1x + \beta_{11}x^2$ .  $MSLF = 62.8154$ ,  $MSPE = 66.0595$ ,  $F^* = 62.8154/66.0595 = 0.95$ ,  $F(.95; 29, 28) = 1.87519$ . If  $F^* \leq 1.87519$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .
- c.  $\hat{Y} = 82.92730 - 1.26789x + .01504x^2 + .000337x^3$
- $H_0: \beta_{111} = 0$ ,  $H_a: \beta_{111} \neq 0$ .  $s\{b_{111}\} = .000933$ ,  $t^* = .000337/.000933 = .3612$ ,  $t(.975; 56) = 2.00324$ . If  $|t^*| \leq 2.00324$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ . Yes. Alternatively,  $SSR(x^3|x, x^2) = 8.6$ ,  $SSE(x, x^2, x^3) = 3662.78$ ,  $F^* = (8.6/1) \div (3662.78/56) = .13148$ ,  $F(.95; 1, 56) = 4.01297$ . If  $F^* \leq 4.01297$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ . Yes.
- 8.19. a.  $\hat{Y} = 2.81311 + 14.3394X_1 - 8.14120X_2 + 1.77739X_1X_2$



- b.  $H_0 : \beta_3 = 0$ ,  $H_a : \beta_3 \neq 0$ .  $s\{b_3\} = .97459$ ,  $t^* = 1.77739/.97459 = 1.8237$ ,  $t(.95; 41) = 1.68288$ . If  $|t^*| \leq 1.68288$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ . Alternatively,  $SSR(X_1X_2|X_1, X_2) = 255.9$ ,  $SSE(X_1, X_2, X_1X_2) = 3154.44$ ,  $F^* = (255.9/1) \div (3154.44/ 41) = 3.32607$ ,  $F(.90; 1, 41) = 2.83208$ . If  $F^* \leq 2.83208$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .

# Chapter 9

## BUILDING THE REGRESSION MODEL I: MODEL SELECTION AND VALIDATION

9.9.

Variables in Model	$R_p^2$	$AIC_p$	$C_p$	$PRESS_p$
None	0	262.916	88.16	13,970.10
$X_1$	.6190	220.529	8.35	5,569.56
$X_2$	.3635	244.131	42.11	9,254.49
$X_3$	.4155	240.214	35.25	8,451.43
$X_1, X_2$	.6550	217.968	5.60	5,235.19
$X_1, X_3$	.6761	215.061	2.81	4,902.75
$X_2, X_3$	.4685	237.845	30.25	8,115.91
$X_1, X_2, X_3$	.6822	216.185	4.00	5,057.886

9.10. b.

$$\begin{matrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{matrix} \begin{bmatrix} 1 & .102 & .181 & .327 \\ & 1 & .519 & .397 \\ & & 1 & .782 \\ & & & 1 \end{bmatrix}$$

c.  $\hat{Y} = -124.3820 + .2957X_1 + .0483X_2 + 1.3060X_3 + .5198X_4$

9.11. a.

Subset	$R_{a,p}^2$
$X_1, X_3, X_4$	.9560
$X_1, X_2, X_3, X_4$	.9555
$X_1, X_3$	.9269
$X_1, X_2, X_3$	.9247

9.17. a.  $X_1, X_3$

b. .10

c.  $X_1, X_3$

d.  $X_1, X_3$

9.18. a.  $X_1, X_3, X_4$

9.21.  $PRESS = 760.974, SSE = 660.657$

9.22. a.

$$\begin{array}{l} X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{bmatrix} 1 & .011 & .177 & .320 \\ & 1 & .344 & .221 \\ & & 1 & .871 \\ & & & 1 \end{bmatrix}$$

b.

	Model-building data set	Validation data set
$b_0$ :	-127.596	-130.652
$s\{b_0\}$ :	12.685	12.189
$b_1$ :	.348	.347
$s\{b_1\}$ :	.054	.048
$b_3$ :	1.823	1.848
$s\{b_3\}$ :	.123	.122
$MSE$ :	27.575	21.446
$R^2$ :	.933	.937

c.  $MSPR = 486.519/25 = 19.461$

d.  $\hat{Y} = -129.664 + .349X_1 + 1.840X_3, s\{b_0\} = 8.445, s\{b_1\} = .035, s\{b_3\} = .084$

# Chapter 10

## BUILDING THE REGRESSION MODEL II: DIAGNOSTICS

10.10.a&f.

$i:$	1	2	...	51	52
$t_i:$	-.224	1.225	...	-1.375	.453
$D_i:$	.0003	.0245	...	.0531	.0015

$t(.9995192; 47) = 3.523$ . If  $|t_i| \leq 3.523$  conclude no outliers, otherwise outliers.  
Conclude no outliers.

b.  $2p/n = 2(4)/52 = .15385$ . Cases 3, 5, 16, 21, 22, 43, 44, and 48.

c.  $\mathbf{X}'_{\text{new}} = [ 1 \quad 300,000 \quad 7.2 \quad 0 ]$

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} 1.8628 & -.0000 & -.1806 & .0473 \\ & .0000 & -.0000 & -.0000 \\ & & .0260 & -.0078 \\ & & & .1911 \end{bmatrix}$$

$h_{\text{new, new}} = .01829$ , no extrapolation

d.

	<i>DFFITs</i>	<i>DFBETAS</i>				<i>D</i>
		$b_0$	$b_1$	$b_2$	$b_3$	
Case 16:	-.554	-.2477	-.0598	.3248	-.4521	.0769
Case 22:	.055	.0304	-.0253	-.0107	.0446	.0008
Case 43:	.562	-.3578	.1338	.3262	.3566	.0792
Case 48:	-.147	.0450	-.0938	.0090	-.1022	.0055
Case 10:	.459	.3641	-.1044	-.3142	-.0633	.0494
Case 32:	-.651	.4095	.0913	-.5708	.1652	.0998
Case 38:	.386	-.0996	-.0827	.2084	-.1270	.0346
Case 40:	.397	.0738	-.2121	.0933	-.1110	.0365

e. Case 16: .161%, case 22: .015%, case 43: .164%, case 48: .042%,  
case 10: .167%, case 32: .227%, case 38: .152%, case 40: .157%.

10.11.a&f.

$i:$	1	2	...	45	46
$t_i:$	.0116	-.9332	...	-.5671	1.0449
$D_i:$	.000003	.015699	...	.006400	.024702

$t(.998913; 41) = 3.27$ . If  $|t_i| \leq 3.27$  conclude no outliers, otherwise outliers. Conclude no outliers.

b.  $2p/n = 2(4)/46 = .1739$ . Cases 9, 28, and 39.

c.  $\mathbf{X}'_{\text{new}} = [ 1 \ 30 \ 58 \ 2.0 ]$

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} 3.24771 & .00922 & -.06793 & -.06730 \\ & .00046 & -.00032 & -.00466 \\ & & .00239 & -.01771 \\ & & & .49826 \end{bmatrix}$$

$h_{\text{new, new}} = .3267$ , extrapolation

d.

	DFFITs	DFBETAS				D
		$b_0$	$b_1$	$b_2$	$b_3$	
Case 11:	.5688	.0991	-.3631	-.1900	.3900	.0766
Case 17:	.6657	-.4491	-.4711	.4432	.0893	.1051
Case 27:	-.6087	-.0172	.4172	-.2499	.1614	.0867

e. Case 11: 1.10%, case 17: 1.32% , case 27: 1.12%.

10.16. b.  $(VIF)_1 = 1.0086$ ,  $(VIF)_2 = 1.0196$ ,  $(VIF)_3 = 1.0144$ .

10.17. b.  $(VIF)_1 = 1.6323$ ,  $(VIF)_2 = 2.0032$ ,  $(VIF)_3 = 2.0091$

10.21. a.  $(VIF)_1 = 1.305$ ,  $(VIF)_2 = 1.300$ ,  $(VIF)_3 = 1.024$

b&c.

$i:$	1	2	3	...	32	33
$e_i:$	13.181	-4.042	3.060	...	14.335	1.396
$e(Y   X_2, X_3):$	26.368	-2.038	-31.111	...	6.310	5.845
$e(X_1   X_2, X_3):$	-.330	-.050	.856	...	.201	.111
$e(Y   X_1, X_3):$	18.734	-17.470	8.212	...	12.566	-8.099
$e(X_2   X_1, X_3):$	-7.537	18.226	-6.993	...	2.401	12.888
$e(Y   X_1, X_2):$	11.542	-7.756	15.022	...	6.732	-15.100
$e(X_3   X_1, X_2):$	-2.111	-4.784	15.406	...	-9.793	-21.247
Exp. value:	11.926	-4.812	1.886	...	17.591	-.940

10.22. a.  $\hat{Y}' = -2.0427 - .7120X'_1 + .7474X'_2 + .7574X'_3$ , where  $Y' = \log_e Y$ ,  $X'_1 = \log_e X_1$ ,  $X'_2 = \log_e(140 - X_2)$ ,  $X'_3 = \log_e X_3$

b.

$i:$	1	2	3	...	31	32	33
$e_i:$	-.0036	.0005	-.0316	...	-.1487	.2863	.1208
Exp. value:	.0238	.0358	-.0481	...	-.1703	.2601	.1164

c.  $(VIF)_1 = 1.339$ ,  $(VIF)_2 = 1.330$ ,  $(VIF)_3 = 1.016$

d&e.

$i:$	1	2	3	...	31	32	33
$h_{ii}:$	.101	.092	.176	...	.058	.069	.149
$t_i:$	-.024	.003	-.218	...	-.975	1.983	.829

$t(.9985; 28) = 3.25$ . If  $|t_i| \leq 3.25$  conclude no outliers, otherwise outliers. Conclude no outliers.

f.

Case	<i>DFFITs</i>	<i>DFBETAS</i>				<i>D</i>
		$b_0$	$b_1$	$b_2$	$b_3$	
28	.739	.530	-.151	-.577	-.187	.120
29	-.719	-.197	-.310	-.133	.420	.109



# Chapter 11

## BUILDING THE REGRESSION MODEL III: REMEDIAL MEASURES

11.7. a.  $\hat{Y} = -5.750 + .1875X$

$i:$	1	2	3	4	5	6
$e_i:$	-3.75	5.75	-13.50	-16.25	-9.75	7.50
$i:$	7	8	9	10	11	12
$e_i:$	-10.50	26.75	14.25	-17.25	-1.75	18.50

b.  $SSR^* = 123,753.125$ ,  $SSE = 2,316.500$ ,

$X_{BP}^2 = (123,753.125/2)/(2,316.500/12)^2 = 1.66$ ,  $\chi^2(.90; 1) = 2.71$ . If  $X_{BP}^2 \leq 2.71$  conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

d.  $\hat{v} = -180.1 + 1.2437X$

$i:$	1	2	3	4	5	6
weight:	.01456	.00315	.00518	.00315	.01456	.00518
$i:$	7	8	9	10	11	12
weight:	.00518	.00315	.01456	.00315	.01456	.00518

e.  $\hat{Y} = -6.2332 + .1891X$

f.

	Unweighted	Weighted
$s\{b_0\}:$	16.7305	13.1672
$s\{b_1\}:$	.0538	.0506

g.  $\hat{Y} = -6.2335 + .1891X$

11.10. a.  $\hat{Y} = 3.32429 + 3.76811X_1 + 5.07959X_2$

d.  $c = .07$

e.  $\hat{Y} = 6.06599 + 3.84335X_1 + 4.68044X_2$

11.11. a.  $\hat{Y} = 1.88602 + 15.1094X$  (47 cases)



$$\hat{Y} = -.58016 + 15.0352X \text{ (45 cases)}$$

b. 
$$\begin{array}{rcccccc} i: & 1 & 2 & \dots & 46 & 47 \\ \hline u_i: & -1.4123 & -.2711 & \dots & 4.6045 & 10.3331 \end{array}$$
  
 smallest weights: .13016 (case 47), .29217 (case 46)

c. 
$$\hat{Y} = -.9235 + 15.13552X$$

d. 2nd iteration: 
$$\hat{Y} = -1.535 + 15.425X$$

3rd iteration: 
$$\hat{Y} = -1.678 + 15.444X$$

smallest weights: .12629 (case 47), .27858 (case 46)

# Chapter 12

## AUTOCORRELATION IN TIME SERIES DATA

12.6.  $H_0 : \rho = 0, H_a : \rho > 0$ .  $D = 2.4015, d_L = 1.29, d_U = 1.38$ . If  $D > 1.38$  conclude  $H_0$ , if  $D < 1.29$  conclude  $H_a$ , otherwise the test is inconclusive. Conclude  $H_0$ .

12.9. a.  $\hat{Y} = -7.7385 + 53.9533X, s\{b_0\} = 7.1746, s\{b_1\} = 3.5197$

$t:$	1	2	3	4	5	6	7	8
$e_t:$	-.0737	-.0709	.5240	.5835	.2612	-.5714	-1.9127	-.8276
$t:$	9	10	11	12	13	14	15	16
$e_t:$	-.6714	.9352	1.803	.4947	.9435	.3156	-.6714	-1.0611

c.  $H_0 : \rho = 0, H_a : \rho > 0$ .  $D = .857, d_L = 1.10, d_U = 1.37$ . If  $D > 1.37$  conclude  $H_0$ , if  $D < 1.10$  conclude  $H_a$ , otherwise the test is inconclusive. Conclude  $H_a$ .

12.10. a.  $r = .5784, 2(1 - .5784) = .8432, D = .857$

b.  $b'_0 = -.69434, b'_1 = 50.93322$

$$\hat{Y}' = -.69434 + 50.93322X'$$

$$s\{b'_0\} = 3.75590, s\{b'_1\} = 4.34890$$

c.  $H_0 : \rho = 0, H_a : \rho > 0$ .  $D = 1.476, d_L = 1.08, d_U = 1.36$ . If  $D > 1.36$  conclude  $H_0$ , if  $D < 1.08$  conclude  $H_a$ , otherwise the test is inconclusive. Conclude  $H_0$ .

d.  $\hat{Y} = -1.64692 + 50.93322X$

$$s\{b_0\} = 8.90868, s\{b_1\} = 4.34890$$

f.  $F_{17} = -1.64692 + 50.93322(2.210) + .5784(-.6595) = 110.534, s\{\text{pred}\} = .9508,$   
 $t(.975; 13) = 2.160, 110.534 \pm 2.160(.9508), 108.48 \leq Y_{17(\text{new})} \leq 112.59$

g.  $t(.975; 13) = 2.160, 50.93322 \pm 2.160(4.349), 41.539 \leq \beta_1 \leq 60.327$ .

12.11. a. 

$\rho:$	.1	.2	.3	.4	.5
$SSE:$	11.5073	10.4819	9.6665	9.0616	8.6710

$\rho:$	.6	.7	.8	.9	1.0
$SSE:$	8.5032	8.5718	8.8932	9.4811	10.3408

$$\rho = .6$$

- b.  $\hat{Y}' = -.5574 + 50.8065X'$ ,  $s\{b'_0\} = 3.5967$ ,  $s\{b'_1\} = 4.3871$
- c.  $H_0 : \rho = 0$ ,  $H_a : \rho > 0$ .  $D = 1.499$ ,  $d_L = 1.08$ ,  $d_U = 1.36$ . If  $D > 1.36$  conclude  $H_0$ , if  $D < 1.08$  conclude  $H_a$ , otherwise test is inconclusive. Conclude  $H_0$ .
- d.  $\hat{Y} = -1.3935 + 50.8065X$ ,  $s\{b_0\} = 8.9918$ ,  $s\{b_1\} = 4.3871$
- f.  $F_{17} = -1.3935 + 50.8065(2.210) + .6(-.6405) = 110.505$ ,  $s\{\text{pred}\} = .9467$ ,  $t(.975; 13) = 2.160$ ,  $110.505 \pm 2.160(.9467)$ ,  $108.46 \leq Y_{17(\text{new})} \leq 112.55$
- 12.12. a.  $b_1 = 49.80564$ ,  $s\{b_1\} = 4.77891$
- b.  $H_0 : \rho = 0$ ,  $H_a : \rho \neq 0$ .  $D = 1.75$  (based on regression with intercept term),  $d_L = 1.08$ ,  $d_U = 1.36$ . If  $D > 1.36$  and  $4 - D > 1.36$  conclude  $H_0$ , if  $D < 1.08$  or  $4 - D < 1.08$  conclude  $H_a$ , otherwise the test is inconclusive. Conclude  $H_0$ .
- c.  $\hat{Y} = .71172 + 49.80564X$ ,  $s\{b_1\} = 4.77891$
- e.  $F_{17} = .71172 + 49.80564(2.210) - .5938 = 110.188$ ,  $s\{\text{pred}\} = .9078$ ,  $t(.975; 14) = 2.145$ ,  $110.188 \pm 2.145(.9078)$ ,  $108.24 \leq Y_{17(\text{new})} \leq 112.14$
- f.  $t(.975; 14) = 2.145$ ,  $49.80564 \pm 2.145(4.77891)$ ,  $39.555 \leq \beta_1 \leq 60.056$

# Chapter 13

## INTRODUCTION TO NONLINEAR REGRESSION AND NEURAL NETWORKS

13.1. a. Intrinsically linear

$$\log_e f(\mathbf{X}, \boldsymbol{\gamma}) = \gamma_0 + \gamma_1 X$$

b. Nonlinear

c. Nonlinear

13.3. b. 300, 3.7323

13.5. a.  $b_0 = -.5072512, b_1 = -0.0006934571, g_0^{(0)} = 0, g_1^{(0)} = .0006934571, g_2^{(0)} = .6021485$

b.  $g_0 = .04823, g_1 = .00112, g_2 = .71341$

13.6. a.  $\hat{Y} = .04823 + .71341\exp(-.00112X)$

City A					
$i:$	1	2	3	4	5
$\hat{Y}_i:$	.61877	.50451	.34006	.23488	.16760
$e_i:$	.03123	-.04451	-.00006	.02512	.00240
Exp. value:	.04125	-.04125	-.00180	.02304	.00180
$i:$	6	7	8		
$\hat{Y}_i:$	.12458	.07320	.05640		
$e_i:$	.02542	-.01320	-.01640		
Exp. value:	.02989	-.01777	-.02304		
City B					
$i:$	9	10	11	12	13
$\hat{Y}_i:$	.61877	.50451	.34006	.23488	.16760
$e_i:$	.01123	-.00451	-.04006	.00512	.02240
Exp. value:	.01327	-.00545	-.02989	.00545	.01777

$i:$	14	15	16
$\hat{Y}_i:$	.12458	.07320	.05640
$e_i:$	-.00458	.00680	-.00640
Exp. value:	-.00923	.00923	-.01327

13.7.  $H_0 : E\{Y\} = \gamma_0 + \gamma_2 \exp(-\gamma_1 X)$ ,  $H_a : E\{Y\} \neq \gamma_0 + \gamma_2 \exp(-\gamma_1 X)$ .

$SSPE = .00290$ ,  $SSE = .00707$ ,  $MSPE = .00290/8 = .0003625$ ,  
 $MSLF = (.00707 - .00290)/5 = .000834$ ,  $F^* = .000834/.0003625 = 2.30069$ ,  
 $F(.99; 5, 8) = 6.6318$ . If  $F^* \leq 6.6318$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

13.8.  $s\{g_0\} = .01456$ ,  $s\{g_1\} = .000092$ ,  $s\{g_2\} = .02277$ ,  $z(.9833) = 2.128$

$$.04823 \pm 2.128(.01456) \qquad .01725 \leq \gamma_0 \leq .07921$$

$$.00112 \pm 2.128(.000092) \qquad .00092 \leq \gamma_1 \leq .00132$$

$$.71341 \pm 2.128(.02277) \qquad .66496 \leq \gamma_2 \leq .76186$$

13.9. a.  $g_0 = .04948$ ,  $g_1 = .00112$ ,  $g_2 = .71341$ ,  $g_3 = -.00250$

b.  $z(.975) = 1.96$ ,  $s\{g_3\} = .01211$ ,  $-.00250 \pm 1.96(.01211)$ ,  $-.02624 \leq \gamma_3 \leq .02124$ ,  
yes, no.

13.13.  $g_0 = 100.3401$ ,  $g_1 = 6.4802$ ,  $g_2 = 4.8155$

13.14. a.  $\hat{Y} = 100.3401 - 100.3401/[1 + (X/4.8155)^{6.4802}]$

b.

$i:$	1	2	3	4	5	6	7
$\hat{Y}_i:$	.0038	.3366	4.4654	11.2653	11.2653	23.1829	23.1829
$e_i:$	.4962	1.9634	-1.0654	.2347	-.3653	.8171	2.1171
Expected Val.:	.3928	1.6354	-1.0519	-.1947	-.5981	.8155	2.0516
$i:$	8	9	10	11	12	13	14
$\hat{Y}_i:$	39.3272	39.3272	56.2506	56.2506	70.5308	70.5308	80.8876
$e_i:$	.2728	-1.4272	-1.5506	.5494	.2692	-2.1308	1.2124
Expected Val.:	.1947	-1.3183	-1.6354	.5981	.0000	-2.0516	1.0519
$i:$	15	16	17	18	19		
$\hat{Y}_i:$	80.8876	87.7742	92.1765	96.7340	98.6263		
$e_i:$	-.2876	1.4258	2.6235	-.5340	-2.2263		
Expected Val.:	-.3928	1.3183	2.7520	-.8155	-2.7520		

13.15.  $H_0 : E\{Y\} = \gamma_0 - \gamma_0/[1 + (X/\gamma_2)^{\gamma_1}]$ ,  $H_a : E\{Y\} \neq \gamma_0 - \gamma_0/[1 + (X/\gamma_2)^{\gamma_1}]$ .

$SSPE = 8.67999$ ,  $SSE = 35.71488$ ,  $MSPE = 8.67999/6 = 1.4467$ ,  $MSLF = (35.71488 - 8.67999)/10 = 2.7035$ ,  $F^* = 2.7035/1.4467 = 1.869$ ,  $F(.99; 10, 6) = 7.87$ . If  $F^* \leq 7.87$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

13.16.  $s\{g_0\} = 1.1741$ ,  $s\{g_1\} = .1943$ ,  $s\{g_2\} = .02802$ ,  $z(.985) = 2.17$

$$100.3401 \pm 2.17(1.1741) \qquad 97.7923 \leq \gamma_0 \leq 102.8879$$

$$6.4802 \pm 2.17(.1943)$$

$$4.8155 \pm 2.17(.02802)$$

$$6.0586 \leq \gamma_1 \leq 6.9018$$

$$4.7547 \leq \gamma_2 \leq 4.8763$$



# Chapter 14

## LOGISTIC REGRESSION, POISSON REGRESSION, AND GENERALIZED LINEAR MODELS

- 14.5. a.  $E\{Y\} = [1 + \exp(-20 + .2X)]^{-1}$   
b. 100  
c.  $X = 125$ :  $\pi = .006692851$ ,  $\pi/(1 - \pi) = .006737947$   
 $X = 126$ :  $\pi = .005486299$ ,  $\pi/(1 - \pi) = .005516565$   
 $005516565/.006737947 = .81873 = \exp(-.2)$
- 14.7. a.  $b_0 = -4.80751$ ,  $b_1 = .12508$ ,  $\hat{\pi} = [1 + \exp(4.80751 - .12508X)]^{-1}$   
c. 1.133  
d. .5487  
e. 47.22
- 14.11. a.
- |         |      |      |      |      |      |      |
|---------|------|------|------|------|------|------|
| $j$ :   | 1    | 2    | 3    | 4    | 5    | 6    |
| $p_j$ : | .144 | .206 | .340 | .592 | .812 | .898 |
- b.  $b_0 = -2.07656$ ,  $b_1 = .13585$   
 $\hat{\pi} = [1 + \exp(2.07656 - .13585X)]^{-1}$   
d. 1.1455  
e. .4903  
f. 23.3726
- 14.14. a.  $b_0 = -1.17717$ ,  $b_1 = .07279$ ,  $b_2 = -.09899$ ,  $b_3 = .43397$   
 $\hat{\pi} = [1 + \exp(1.17717 - .07279X_1 + .09899X_2 - .43397X_3)]^{-1}$   
b.  $\exp(b_1) = 1.0755$ ,  $\exp(b_2) = .9058$ ,  $\exp(b_3) = 1.5434$   
c. .0642
- 14.15. a.  $z(.95) = 1.645$ ,  $s\{b_1\} = .06676$ ,  $\exp[.12508 \pm 1.645(.06676)]$ ,



$$1.015 \leq \exp(\beta_1) \leq 1.265$$

- b.  $H_0 : \beta_1 = 0, H_a : \beta_1 \neq 0. b_1 = .12508, s\{b_1\} = .06676, z^* = .12508/.06676 = 1.8736. z(.95) = 1.645, |z^*| \leq 1.645, \text{conclude } H_0, \text{ otherwise conclude } H_a. \text{ Conclude } H_a. P\text{-value}=.0609.$
- c.  $H_0 : \beta_1 = 0, H_a : \beta_1 \neq 0. G^2 = 3.99, \chi^2(.90; 1) = 2.7055. \text{ If } G^2 \leq 2.7055, \text{ conclude } H_0, \text{ otherwise conclude } H_a. \text{ Conclude } H_a. P\text{-value}=.046$
- 14.17. a.  $z(.975) = 1.960, s\{b_1\} = .004772, .13585 \pm 1.960(.004772),$   
 $.1265 \leq \beta_1 \leq .1452, \quad 1.1348 \leq \exp(\beta_1) \leq 1.1563.$
- b.  $H_0 : \beta_1 = 0, H_a : \beta_1 \neq 0. b_1 = .13585, s\{b_1\} = .004772, z^* = .13585/.004772 = 28.468. z(.975) = 1.960, |z^*| \leq 1.960, \text{conclude } H_0, \text{ otherwise conclude } H_a. \text{ Conclude } H_a. P\text{-value} = 0+.$
- c.  $H_0 : \beta_1 = 0, H_a : \beta_1 \neq 0. G^2 = 1095.99, \chi^2(.95; 1) = 3.8415. \text{ If } G^2 \leq 3.8415, \text{ conclude } H_0, \text{ otherwise conclude } H_a. \text{ Conclude } H_a. P\text{-value} = 0+.$
- 14.20. a.  $z(1-.1/[2(2)]) = z(.975) = 1.960, s\{b_1\} = .03036, s\{b_2\} = .03343, \exp\{30[.07279 \pm 1.960(.03036)]\}, 1.49 \leq \exp(30\beta_1) \leq 52.92, \exp\{25[-.09899 \pm 1.960(.03343)]\}, .016 \leq \exp(2\beta_2) \leq .433.$
- b.  $H_0 : \beta_3 = 0, H_a : \beta_3 \neq 0. b_3 = .43397, s\{b_3\} = .52132, z^* = .43397/.52132 = .8324. z(.975) = 1.96, |z^*| \leq 1.96, \text{conclude } H_0, \text{ otherwise conclude } H_a. \text{ Conclude } H_0. P\text{-value} = .405.$
- c.  $H_0 : \beta_3 = 0, H_a : \beta_3 \neq 0. G^2 = .702, \chi^2(.95; 1) = 3.8415. \text{ If } G^2 \leq 3.8415, \text{ conclude } H_0, \text{ otherwise conclude } H_a. \text{ Conclude } H_0.$
- d.  $H_0 : \beta_3 = \beta_4 = \beta_5 = 0, H_a : \text{not all } \beta_k = 0, \text{ for } k = 3, 4, 5. G^2 = 1.534, \chi^2(.95; 3) = 7.81. \text{ If } G^2 \leq 7.81, \text{ conclude } H_0, \text{ otherwise conclude } H_a. \text{ Conclude } H_0.$
- 14.22. a.  $X_1$  enters in step 1;  $X_2$  enters in step 2;  
no variables satisfy criterion for entry in step 3.
- b.  $X_{11}$  is deleted in step 1;  $X_{12}$  is deleted in step 2;  $X_3$  is deleted in step 3;  $X_{22}$  is deleted in step 4;  $X_1$  and  $X_2$  are retained in the model.
- c. The best model according to the  $AIC_p$  criterion is based on  $X_1$  and  $X_2. AIC_3 = 111.795.$
- d. The best model according to the  $SBC_p$  criterion is based on  $X_1$  and  $X_2. SBC_3 = 121.002.$

14.23.

$j:$	1	2	3	4	5	6
$O_{j1}:$	72	103	170	296	406	449
$E_{j1}:$	71.0	99.5	164.1	327.2	394.2	440.0
$O_{j0}:$	428	397	330	204	94	51
$E_{j0}:$	429.0	400.5	335.9	172.9	105.8	60.0

$$H_0 : E\{Y\} = [1 + \exp(-\beta_0 - \beta_1 X)]^{-1},$$

$$H_a : E\{Y\} \neq [1 + \exp(-\beta_0 - \beta_1 X)]^{-1}.$$

$X^2 = 12.284$ ,  $\chi^2(.99; 4) = 13.28$ . If  $X^2 \leq 13.28$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

14.25. a.

Class $j$	$\hat{\pi}'$ Interval	Midpoint	$n_j$	$p_j$
1	-1.1 - under -.4	-.75	10	.3
2	-.4 - under .6	.10	10	.6
3	.6 - under 1.5	1.05	10	.7

b.

$i$ :	1	2	3	...	28	29	30
$r_{SP_i}$ :	-.6233	1.7905	-.6233	...	.6099	.5754	-2.0347

14.28. a.

$j$ :	1	2	3	4	5	6	7	8
$O_{j1}$ :	0	1	0	2	1	8	2	10
$E_{j1}$ :	.2	.5	1.0	1.5	2.4	3.4	4.7	10.3
$O_{j0}$ :	19	19	20	18	19	12	18	10
$E_{j0}$ :	18.8	19.5	19.0	18.5	17.6	16.6	15.3	9.7

b.  $H_0 : E\{Y\} = [1 + \exp(-\beta_0 - \beta_1 X_1 - \beta_2 X_2 - \beta_3 X_3)]^{-1},$

$$H_a : E\{Y\} \neq [1 + \exp(-\beta_0 - \beta_1 X_1 - \beta_2 X_2 - \beta_3 X_3)]^{-1}.$$

$X^2 = 12.116$ ,  $\chi^2(.95; 6) = 12.59$ . If  $X^2 \leq 12.59$ , conclude  $H_0$ , otherwise conclude  $H_a$ . Conclude  $H_0$ .  $P$ -value = .0594.

c.

$i$ :	1	2	3	...	157	158	159
$dev_i$ :	-.5460	-.5137	1.1526	...	.4248	.8679	1.6745

14.29 a.

$i$ :	1	2	3	...	28	29	30
$h_{ii}$ :	.1040	.1040	.1040	...	.0946	.1017	.1017

b.

$i$ :	1	2	3	...	28	29	30
$\Delta X_i^2$ :	.3885	3.2058	.3885	...	4.1399	.2621	.2621
$\Delta dev_i$ :	.6379	3.0411	.6379	...	3.5071	.4495	.4495
$D_i$ :	.0225	.1860	.0225	...	.2162	.0148	.0148

14.32 a.

$i$ :	1	2	3	...	157	158	159
$h_{ii}$ :	.0197	.0186	.0992	...	.0760	.1364	.0273

b.

$i:$	1	2	3	...	157	158	159
$\Delta X_i^2:$	.1340	.1775	1.4352	...	.0795	.6324	2.7200
$\Delta dev_i:$	.2495	.3245	1.8020	...	.1478	.9578	2.6614
$D_i:$	.0007	.0008	.0395	...	.0016	.0250	.0191

14.33. a.  $z(.95) = 1.645$ ,  $\hat{\pi}'_h = .19561$ ,  $s^2\{b_0\} = 7.05306$ ,  $s^2\{b_1\} = .004457$ ,  $s\{b_0, b_1\} = -.175353$ ,  $s\{\hat{\pi}'_h\} = .39428$ ,  $.389 \leq \pi_h \leq .699$

b.

Cutoff	Renewers	Nonrenewers	Total
.40	18.8	50.0	33.3
.45	25.0	50.0	36.7
.50	25.0	35.7	30.0
.55	43.8	28.6	36.7
.60	43.8	21.4	33.3

c. Cutoff = .50. Area = .70089.

14.36. a.  $\hat{\pi}'_h = -1.3953$ ,  $s^2\{\hat{\pi}'_h\} = .1613$ ,  $s\{\hat{\pi}'_h\} = .4016$ ,  $z(.95) = 1.645$ .  $L = -1.3953 - 1.645(.4016) = -2.05597$ ,  $U = -1.3953 + 1.645(.4016) = -.73463$ .  
 $L^* = [1 + \exp(2.05597)]^{-1} = .11345$ ,  $U^* = [1 + \exp(.73463)]^{-1} = .32418$ .

b.

Cutoff	Received	Not receive	Total
.05	4.35	62.20	66.55
.10	13.04	39.37	52.41
.15	17.39	26.77	44.16
.20	39.13	15.75	54.88

c. Cutoff = .15. Area = .82222.

14.38. a.  $b_0 = 2.3529$ ,  $b_1 = .2638$ ,  $s\{b_0\} = .1317$ ,  $s\{b_1\} = .0792$ ,  $\hat{\mu} = \exp(2.3529 + .2638X)$ .

b.

$i:$	1	2	3	...	8	9	10
$dev_i:$	.6074	-.4796	-.1971	...	.3482	.2752	.1480

c.

$X_h:$	0	1	2	3
Poisson:	10.5	13.7	17.8	23.2
Linear:	10.2	14.2	18.2	22.2

e.  $\hat{\mu}_h = \exp(2.3529) = 10.516$

$$P(Y \leq 10 | X_h = 0) = \sum_{Y=0}^{10} \frac{(10.516)^Y \exp(-10.516)}{Y!}$$

$$= 2.7 \times 10^{-5} + \dots + .1235 = .5187$$

f.  $z(.975) = 1.96$ ,  $.2638 \pm 1.96(.0792)$ ,  $.1086 \leq \beta_1 \leq .4190$

# Chapter 15

## INTRODUCTION TO THE DESIGN OF EXPERIMENTAL AND OBSERVATIONAL STUDIES

- 15.9. a. Observational.  
b. Factor: expenditures for research and development in the past three years.  
Factor levels: low, moderate, and high.  
c. Cross-sectional study.  
d. Firm.
- 15.14. a. Experimental.  
b. Factor 1: ingredient 1, with three levels (low, medium, high).  
Factor 2: ingredient 2, with three levels (low, medium, high).  
There are 9 factor-level combinations.  
d. Completely randomized design.  
e. Volunteer.
- 15.20. a.  $2^3$  factorial design with two replicates.  
c. Rod.
- 15.23. a.  $H_0: \bar{W} = 0, H_a: \bar{W} \neq 0. t^* = -.1915/.0112 = -17.10, t(.975, 19) = 2.093.$  If  $|t^*| > 2.093$  conclude  $H_0$ , otherwise conclude  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+.  
Agree with results on page 670. They should agree.  
b.  $H_0: \beta_2 = \dots = \beta_{20} = 0, H_a: \text{not all } \beta_k (k = 2, 3, \dots, 20) \text{ equal zero. } F^* = [(.23586 - .023828)/(38 - 19)] \div [.023828/19] = 8.90, F(.95; 19, 19) = 2.17.$  If  $F^* > 2.17$  conclude  $H_0$ , otherwise conclude  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+.  
Not of primary interest because blocking factor was used here to increase the precision.



# Chapter 16

## SINGLE-FACTOR STUDIES

16.7. b.  $\hat{Y}_{1j} = \bar{Y}_1 = 6.87778$ ,  $\hat{Y}_{2j} = \bar{Y}_2 = 8.13333$ ,  $\hat{Y}_{3j} = \bar{Y}_3 = 9.20000$

c.  $e_{ij}$ :

$i$	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$
1	.772	1.322	-.078	-1.078	.022	-.278
2	-1.433	-.033	1.267	.467	-.333	-.433
3	-.700	.500	.900	-1.400	.400	.300

  

$i$	$j = 7$	$j = 8$	$j = 9$	$j = 10$	$j = 11$	$j = 12$
1	-.578	.822	-.878			
2	.767	-.233	.167	.567	-1.033	.267

Yes

d.

Source	$SS$	$df$	$MS$
Between levels	20.125	2	10.0625
Error	15.362	24	.6401
Total	35.487	26	

e.  $H_0$ : all  $\mu_i$  are equal ( $i = 1, 2, 3$ ),  $H_a$ : not all  $\mu_i$  are equal.  $F^* = 10.0625/.6401 = 15.720$ ,  $F(.95; 2, 24) = 3.40$ . If  $F^* \leq 3.40$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .

f.  $P$ -value = 0+

16.10. b.  $\hat{Y}_{1j} = \bar{Y}_1 = 21.500$ ,  $\hat{Y}_{2j} = \bar{Y}_2 = 27.750$ ,  $\hat{Y}_{3j} = \bar{Y}_3 = 21.417$

c.  $e_{ij}$ :

$i$	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$
1	1.500	3.500	-.500	.500	-.500	.500
2	.250	-.750	-.750	1.250	-1.750	1.250
3	1.583	-1.417	3.583	-.417	.583	1.583

  

$i$	$j = 7$	$j = 8$	$j = 9$	$j = 10$	$j = 11$	$j = 12$
1	-1.500	1.500	-2.500	.500	-2.500	-.500
2	-.750	2.250	.250	-.750	-1.750	1.250
3	-.417	-1.417	-2.417	-1.417	.583	-.417

d.

Source	<i>SS</i>	<i>df</i>	<i>MS</i>
Between ages	316.722	2	158.361
Error	82.167	33	2.490
Total	398.889	35	

e.  $H_0$ : all  $\mu_i$  are equal ( $i = 1, 2, 3$ ),  $H_a$ : not all  $\mu_i$  are equal.  $F^* = 158.361/2.490 = 63.599$ ,  $F(.99; 2, 33) = 5.31$ . If  $F^* \leq 5.31$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

16.11. b.  $\hat{Y}_{1j} = \bar{Y}_1 = .0735$ ,  $\hat{Y}_{2j} = \bar{Y}_2 = .1905$ ,  $\hat{Y}_{3j} = \bar{Y}_3 = .4600$ ,  $\hat{Y}_{4j} = \bar{Y}_4 = .3655$ ,  
 $\hat{Y}_{5j} = \bar{Y}_5 = .1250$ ,  $\hat{Y}_{6j} = \bar{Y}_6 = .1515$

c.  $e_{ij}$ :

<i>i</i>	<i>j</i> = 1	<i>j</i> = 2	<i>j</i> = 3	<i>j</i> = 4	<i>j</i> = 5
1	-.2135	.1265	-.0035	.1065	.3065
2	.2695	-.0805	-.0705	.2795	.0495
3	-.2500	.3200	-.1400	-.0100	-.2400
4	.1245	.2145	.1545	-.0755	-.0955
5	-.3150	.1450	-.0650	-.0150	.1050
6	-.1015	-.2015	.1285	.3185	-.0315

<i>i</i>	<i>j</i> = 6	<i>j</i> = 7	<i>j</i> = 8	<i>j</i> = 9	<i>j</i> = 10
1	.0265	-.1135	-.3435	.1965	-.2835
2	-.1305	-.3105	.1395	-.1305	-.2205
3	-.1100	.0800	-.2200	.0100	.1600
4	.1845	.0345	-.2255	.1145	-.0255
5	.0250	-.1150	.0950	.1650	.0150
6	.1185	-.0715	.0185	.2785	-.2215

<i>i</i>	<i>j</i> = 11	<i>j</i> = 12	<i>j</i> = 13	<i>j</i> = 14	<i>j</i> = 15
1	.3165	-.1435	-.0935	.2065	.0165
2	-.1405	.3395	.2295	.0995	.1695
3	.0100	.0900	.1300	.2500	-.0100
4	-.3555	-.0355	-.1855	-.2355	.1145
5	.0750	.1750	-.2350	.1450	-.3250
6	.0485	-.1415	-.0515	.0085	-.2115

<i>i</i>	<i>j</i> = 16	<i>j</i> = 17	<i>j</i> = 18	<i>j</i> = 19	<i>j</i> = 20
1	.0565	.1865	-.0035	-.0835	-.2635
2	-.1505	-.0205	-.1705	-.0805	-.0705
3	.0200	-.0200	.0400	-.2600	.1500
4	.1745	.1445	.0545	.0845	-.1655
5	.1150	.0750	.0150	.2250	-.3050
6	-.0215	.2785	.1985	-.2415	-.1015

Yes

d.

Source	<i>SS</i>	<i>df</i>	<i>MS</i>
Between machines	2.28935	5	.45787
Error	3.53060	114	.03097
Total	5.81995	119	

- e.  $H_0$ : all  $\mu_i$  are equal ( $i = 1, \dots, 6$ ),  $H_a$ : not all  $\mu_i$  are equal.  $F^* = .45787/.03097 = 14.78$ ,  $F(.95; 5, 114) = 2.29$ . If  $F^* \leq 2.29$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .
- f.  $P$ -value = 0+

16.18. a.

$$\mathbf{Y} = \begin{bmatrix} 7.6 \\ 8.2 \\ 6.8 \\ 5.8 \\ 6.9 \\ 6.6 \\ 6.3 \\ 7.7 \\ 6.0 \\ 6.7 \\ 8.1 \\ 9.4 \\ 8.6 \\ 7.8 \\ 7.7 \\ 8.9 \\ 7.9 \\ 8.3 \\ 8.7 \\ 7.1 \\ 8.4 \\ 8.5 \\ 9.7 \\ 10.1 \\ 7.8 \\ 9.6 \\ 9.5 \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix} \quad \boldsymbol{\beta} = \begin{bmatrix} \mu. \\ \tau_1 \\ \tau_2 \end{bmatrix}$$

b.



$$\mathbf{X}\boldsymbol{\beta} = \begin{bmatrix} \mu. + \tau_1 \\ \mu. + \tau_1 \\ \mu. + \tau_1 \\ \mu. + \tau_1 \\ \mu. + \tau_1 \\ \mu. + \tau_1 \\ \mu. + \tau_1 \\ \mu. + \tau_1 \\ \mu. + \tau_1 \\ \mu. + \tau_1 \\ \mu. + \tau_2 \\ \mu. + \tau_2 \\ \mu. + \tau_2 \\ \mu. + \tau_2 \\ \mu. + \tau_2 \\ \mu. + \tau_2 \\ \mu. + \tau_2 \\ \mu. + \tau_2 \\ \mu. + \tau_2 \\ \mu. + \tau_2 \\ \mu. + \tau_2 \\ \mu. + \tau_2 \\ \mu. + \tau_2 \\ \mu. + \tau_2 \\ \mu. - \tau_1 - \tau_2 \\ \mu. - \tau_1 - \tau_2 \\ \mu. - \tau_1 - \tau_2 \\ \mu. - \tau_1 - \tau_2 \\ \mu. - \tau_1 - \tau_2 \\ \mu. - \tau_1 - \tau_2 \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_3 \\ \mu_3 \\ \mu_3 \\ \mu_3 \\ \mu_3 \end{bmatrix}$$

c.  $\hat{Y} = 8.07037 - 1.19259X_1 + .06296X_2$ ,  $\mu.$  defined in (16.63)

d.

Source	<i>SS</i>	<i>df</i>	<i>MS</i>
Regression	20.125	2	10.0625
Error	15.362	24	.6401
Total	35.487	26	

e.  $H_0: \tau_1 = \tau_2 = 0$ ,  $H_a$ : not both  $\tau_1$  and  $\tau_2$  equal zero.  $F^* = 10.0625/.6401 = 15.720$ ,  $F(.95; 2, 24) = 3.40$ . If  $F^* \leq 3.40$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .

16.21. a.  $\hat{Y} = 23.55556 - 2.05556X_1 + 4.19444X_2$ ,  $\mu.$  defined in (16.63)

b.

Source	<i>SS</i>	<i>df</i>	<i>MS</i>
Regression	316.722	2	158.361
Error	82.167	33	2.490
Total	398.889	35	

$H_0: \tau_1 = \tau_2 = 0$ ,  $H_a$ : not both  $\tau_1$  and  $\tau_2$  equal zero.  $F^* = 158.361/2.490 = 63.599$ ,  $F(.99; 2, 33) = 5.31$ . If  $F^* \leq 5.31$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .

16.25.  $\mu. = 7.889, \phi = 2.457, 1 - \beta \cong .95$

16.27  $\mu. = 24, \phi = 6.12, 1 - \beta > .99$

16.29 a.

$$\frac{\Delta : 10 \quad 15 \quad 20 \quad 30}{n : 51 \quad 23 \quad 14 \quad 7}$$

b.

$$\frac{\Delta : 10 \quad 15 \quad 20 \quad 30}{n : 39 \quad 18 \quad 11 \quad 6}$$

16.34. a.  $\Delta/\sigma = .15/.15 = 1.0, n = 22$

b.  $\phi = \frac{1}{.15} \left[ \frac{22}{6} (.02968) \right]^{1/2} = 2.199, 1 - \beta \geq .97$

c.  $(.10\sqrt{n})/.15 = 3.1591, n = 23$



# Chapter 17

## ANALYSIS OF FACTOR LEVEL MEANS

- 17.8. a.  $\bar{Y}_1 = 6.878, \bar{Y}_2 = 8.133, \bar{Y}_3 = 9.200$   
 b.  $s\{\bar{Y}_3\} = .327, t(.975; 24) = 2.064, 9.200 \pm 2.064(.327), 8.525 \leq \mu_3 \leq 9.875$   
 c.  $\hat{D} = \bar{Y}_2 - \bar{Y}_1 = 1.255, s\{\hat{D}\} = .353, t(.975; 24) = 2.064, 1.255 \pm 2.064(.353), .526 \leq D \leq 1.984$   
 d.  $\hat{D}_1 = \bar{Y}_3 - \bar{Y}_2 = 1.067, \hat{D}_2 = \bar{Y}_3 - \bar{Y}_1 = 2.322, \hat{D}_3 = \bar{Y}_2 - \bar{Y}_1 = 1.255, s\{\hat{D}_1\} = .400, s\{\hat{D}_2\} = .422, s\{\hat{D}_3\} = .353, q(.90; 3, 24) = 3.05, T = 2.157$   
 $1.067 \pm 2.157(.400) \quad .204 \leq D_1 \leq 1.930$   
 $2.322 \pm 2.157(.422) \quad 1.412 \leq D_2 \leq 3.232$   
 $1.255 \pm 2.157(.353) \quad .494 \leq D_3 \leq 2.016$   
 e.  $F(.90; 2, 24) = 2.54, S = 2.25$   
 $B = t(.9833; 24) = 2.257$   
 Yes
- 17.11. a.  $\bar{Y}_1 = 21.500, \bar{Y}_2 = 27.750, \bar{Y}_3 = 21.417$   
 b.  $MSE = 2.490, s\{\bar{Y}_1\} = .456, t(.995; 33) = 2.733, 21.500 \pm 2.733(.456), 20.254 \leq \mu_1 \leq 22.746$   
 c.  $\hat{D} = \bar{Y}_3 - \bar{Y}_1 = -.083, s\{\hat{D}\} = .644, t(.995; 33) = 2.733, -.083 \pm 2.733(.644), -1.843 \leq D \leq 1.677$   
 d.  $H_0 : 2\mu_2 - \mu_1 - \mu_3 = 0, H_a : 2\mu_2 - \mu_1 - \mu_3 \neq 0. F^* = (12.583)^2/1.245 = 127.17, F(.99; 1, 33) = 7.47. \text{ If } F^* \leq 7.47 \text{ conclude } H_0, \text{ otherwise } H_a. \text{ Conclude } H_a.$   
 e.  $\hat{D}_1 = \bar{Y}_3 - \bar{Y}_1 = -.083, \hat{D}_2 = \bar{Y}_3 - \bar{Y}_2 = -6.333, \hat{D}_3 = \bar{Y}_2 - \bar{Y}_1 = 6.250, s\{\hat{D}_i\} = .644 (i = 1, 2, 3), q(.90; 3, 33) = 3.01, T = 2.128$   
 $-.083 \pm 2.128(.644) \quad -1.453 \leq D_1 \leq 1.287$   
 $-6.333 \pm 2.128(.644) \quad -7.703 \leq D_2 \leq -4.963$   
 $6.250 \pm 2.128(.644) \quad 4.880 \leq D_3 \leq 7.620$   
 f.  $B = t(.9833; 33) = 2.220, \text{ no}$
- 17.12. a.  $\bar{Y}_1 = .0735, \bar{Y}_2 = .1905, \bar{Y}_3 = .4600, \bar{Y}_4 = .3655, \bar{Y}_5 = .1250, \bar{Y}_6 = .1515$

- b.  $MSE = .03097$ ,  $s\{\bar{Y}_1\} = .0394$ ,  $t(.975; 114) = 1.981$ ,  $.0735 \pm 1.981(.0394)$ ,  
 $-.005 \leq \mu_1 \leq .152$
- c.  $\hat{D} = \bar{Y}_2 - \bar{Y}_1 = .1170$ ,  $s\{\hat{D}\} = .0557$ ,  $t(.975; 114) = 1.981$ ,  $.1170 \pm 1.981(.0557)$ ,  
 $.007 \leq D \leq .227$
- e.  $\hat{D}_1 = \bar{Y}_1 - \bar{Y}_4 = -.2920$ ,  $\hat{D}_2 = \bar{Y}_1 - \bar{Y}_5 = -.0515$ ,  $\hat{D}_3 = \bar{Y}_4 - \bar{Y}_5 = .2405$ ,  
 $s\{\hat{D}_i\} = .0557$  ( $i = 1, 2, 3$ ),  $B = t(.9833; 114) = 2.178$

Test Comparison

$i$	$i$	$t_i^*$	Conclusion
1	$D_1$	-5.242	$H_a$
2	$D_2$	-.925	$H_0$
3	$D_3$	4.318	$H_a$

- f.  $q(.90; 6, 114) = 3.71$ ,  $T = 2.623$ , no

- 17.14. a.  $\hat{L} = (\bar{Y}_1 + \bar{Y}_2)/2 - \bar{Y}_3 = (6.8778 + 8.1333)/2 - 9.200 = -1.6945$ ,  
 $s\{\hat{L}\} = .3712$ ,  $t(.975; 24) = 2.064$ ,  $-1.6945 \pm 2.064(.3712)$ ,  $-2.461 \leq L \leq -.928$
- b.  $\hat{L} = (3/9)\bar{Y}_1 + (4/9)\bar{Y}_2 + (2/9)\bar{Y}_3 = 7.9518$ ,  $s\{\hat{L}\} = .1540$ ,  $t(.975; 24) = 2.064$ ,  
 $7.9518 \pm 2.064(.1540)$ ,  $7.634 \leq L \leq 8.270$
- c.  $F(.90; 2, 24) = 2.54$ ,  $S = 2.254$ ; see also part (a) and Problem 17.8.

$$1.067 \pm 2.254(.400) \quad .165 \leq D_1 \leq 1.969$$

$$2.322 \pm 2.254(.422) \quad 1.371 \leq D_2 \leq 3.273$$

$$1.255 \pm 2.254(.353) \quad .459 \leq D_3 \leq 2.051$$

$$-1.6945 \pm 2.254(.3712) \quad -2.531 \leq L_1 \leq -.858$$

- 17.16. a.  $\hat{L} = (\bar{Y}_3 - \bar{Y}_2) - (\bar{Y}_2 - \bar{Y}_1) = \bar{Y}_3 - 2\bar{Y}_2 + \bar{Y}_1 = 21.4167 -$   
 $2(27.7500) + 21.500 = -12.5833$ ,  $s\{\hat{L}\} = 1.1158$ ,  $t(.995; 33) = 2.733$ ,  
 $-12.5833 \pm 2.733(1.1158)$ ,  $-15.632 \leq L \leq -9.534$
- b.  $\hat{D}_1 = \bar{Y}_2 - \bar{Y}_1 = 6.2500$ ,  $\hat{D}_2 = \bar{Y}_3 - \bar{Y}_2 = -6.3333$ ,  $\hat{D}_3 = \bar{Y}_3 - \bar{Y}_1 = -.0833$ ,  
 $\hat{L}_1 = \hat{D}_2 - \hat{D}_1 = -12.5833$ ,  $s\{\hat{D}_i\} = .6442$  ( $i = 1, 2, 3$ ),  $s\{\hat{L}_1\} = 1.1158$ ,  
 $F(.90; 2, 33) = 2.47$ ,  $S = 2.223$

$$6.2500 \pm 2.223(.6442) \quad 4.818 \leq D_1 \leq 7.682$$

$$-6.3333 \pm 2.223(.6442) \quad -7.765 \leq D_2 \leq -4.901$$

$$-.0833 \pm 2.223(.6442) \quad -1.515 \leq D_3 \leq 1.349$$

$$-12.5833 \pm 2.223(1.1158) \quad -15.064 \leq L_1 \leq -10.103$$

- 17.17. a.  $\hat{L} = (\bar{Y}_1 + \bar{Y}_2)/2 - (\bar{Y}_3 + \bar{Y}_4)/2 = (.0735 + .1905)/2 - (.4600 + .3655)/2$   
 $= -.28075$ ,  $s\{\hat{L}\} = .03935$ ,  $t(.975; 114) = 1.981$ ,  $-.28075 \pm 1.981(.03935)$ ,  $-.3587 \leq$   
 $L \leq -.2028$

- b.  $\hat{D}_1 = -.1170$ ,  $\hat{D}_2 = .0945$ ,  $\hat{D}_3 = -.0265$ ,  $\hat{L}_1 = -.28075$ ,  $\hat{L}_2 = -.00625$ ,  $\hat{L}_3 =$   
 $-.2776$ ,  $\hat{L}_4 = .1341$ ,  $s\{\hat{D}_i\} = .0557$  ( $i = 1, 2, 3$ ),  $s\{\hat{L}_1\} = s\{\hat{L}_2\} = .03935$ ,  
 $s\{\hat{L}_3\} = s\{\hat{L}_4\} = .03408$ ,  $B = t(.99286; 114) = 2.488$
- $$-.1170 \pm 2.488(.0557) \quad -.2556 \leq D_1 \leq .0216$$

$$\begin{array}{ll}
.0945 \pm 2.488(.0557) & -.0441 \leq D_2 \leq .2331 \\
-.0265 \pm 2.488(.0557) & -.1651 \leq D_3 \leq .1121 \\
-.28075 \pm 2.488(.03935) & -.3787 \leq L_1 \leq -.1828 \\
-.00625 \pm 2.488(.03935) & -.1042 \leq L_2 \leq .0917 \\
-.2776 \pm 2.488(.03408) & -.3624 \leq L_3 \leq -.1928 \\
.1341 \pm 2.488(.03408) & .0493 \leq L_4 \leq .2189
\end{array}$$

17.19. a.  $L_1 = \mu_1 - \mu.$      $L_2 = \mu_2 - \mu.$

$L_3 = \mu_3 - \mu.$      $L_4 = \mu_4 - \mu.$

$L_5 = \mu_5 - \mu.$      $L_6 = \mu_6 - \mu.$

$\hat{L}_1 = .0735 - .2277 = -.1542, \hat{L}_2 = .1905 - .2277 = -.0372$

$\hat{L}_3 = .4600 - .2277 = .2323, \hat{L}_4 = .3655 - .2277 = .1378$

$\hat{L}_5 = .1250 - .2277 = -.1027, \hat{L}_6 = .1515 - .2277 = -.0762$

$$s\{\hat{L}_i\} = \sqrt{\frac{.03097}{20} \left(\frac{25}{36}\right) + \frac{.03097}{36} \left(\frac{5}{20}\right)} = .0359$$

$B = t(.99583; 114) = 2.685$

$-.1542 \pm 2.685(.0359)$        $.2506 \leq L_1 \leq -.0578$

$-.0372 \pm 2.685(.0359)$        $-.1336 \leq L_2 \leq .0592$

$.2323 \pm 2.685(.0359)$        $.1359 \leq L_3 \leq .3287$

$.1378 \pm 2.685(.0359)$        $.0414 \leq L_4 \leq .2342$

$-.1027 \pm 2.685(.0359)$        $-.1991 \leq L_5 \leq -.0063$

$-.0762 \pm 2.685(.0359)$        $-.1726 \leq L_6 \leq .0202$

Conclude not all  $\mu_i$  are equal.

17.25. Bonferroni,  $n = 45$

17.29. a.  $\hat{Y} = .18472 + .06199x + .01016x^2$

b.  $e_{ij}$  :

$i$	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$	$j = 7$
1	-.2310	.1090	-.0210	.0890	.2890	.0090	-.1310
2	.2393	-.1107	-.1007	.2493	.0193	-.1607	-.3407
3	-.2440	.3260	-.1340	-.0040	-.2340	-.1040	.0860
4	.1268	.2168	.1568	-.0732	-.0932	.1868	.0368
5	-.2969	.1631	-.0469	.0031	.1231	.0431	-.0969
6	-.0802	-.1802	.1498	.3398	-.0102	.1398	-.0502
$i$	$j = 8$	$j = 9$	$j = 10$	$j = 11$	$j = 12$	$j = 13$	$j = 14$
1	-.3610	.1790	-.3010	.2990	-.1610	-.1110	.1890
2	.1093	-.1607	-.2507	-.1707	.3093	.1993	.0693
3	-.2140	.0160	.1660	.0160	.0960	.1360	.2560
4	-.2232	.1168	-.0232	-.3532	-.0332	-.1832	-.2332
5	.1131	.1831	.0331	.0931	.1931	-.2169	.1631
6	.0398	.2998	-.2002	.0698	-.1202	-.0302	.0298

$i$	$j = 15$	$j = 16$	$j = 17$	$j = 18$	$j = 19$	$j = 20$
1	-.0010	.0390	.1690	-.0210	-.1010	-.2810
2	.1393	-.1807	-.0507	-.2007	-.1107	-.1007
3	-.0040	.0260	-.0140	.0460	-.2540	.1560
4	.1168	.1768	.1468	.0568	.0868	-.1632
5	-.3069	.1331	.0931	.0331	.2431	-.2869
6	-.1902	-.0002	.2998	.2198	-.2202	-.0802

- c.  $H_0 : E\{Y\} = \beta_0 + \beta_1x + \beta_{11}x^2$ ,  $H_a : E\{Y\} \neq \beta_0 + \beta_1x + \beta_{11}x^2$ .  $SSPE = 3.5306$ ,  $SSLF = .0408$ ,  $F^* = (.0408/3) \div (3.5306/114) = .439$ ,  $F(.99; 3, 114) = 3.96$ . If  $F^* \leq 3.96$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .
- d.  $H_0 : \beta_{11} = 0$ ,  $H_a : \beta_{11} \neq 0$ .  $s\{b_{11}\} = .00525$ ,  $t^* = .01016/.00525 = 1.935$ ,  $t(.995; 117) = 2.619$ . If  $|t^*| \leq 2.619$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

# Chapter 18

## ANOVA DIAGNOSTICS AND REMEDIAL MEASURES

18.4. a. See Problem 16.7c.

b.  $r = .992$

c.  $t_{ij}$ :

$i$	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$
1	.9557	1.8377	-.1010	-1.4623	.0288	-.3615
2	-1.9821	-.0426	1.7197	.6011	-.4277	-.5575
3	-.9568	.6768	1.2464	-2.0391	.5395	.4035

  

$i$	$j = 7$	$j = 8$	$j = 9$	$j = 10$	$j = 11$	$j = 12$
1	-.7592	1.0945	-1.1728			
2	1.0009	-.2988	.2132	.7326	-1.3737	.3417

$H_0$ : no outliers,  $H_a$ : at least one outlier.  $t(.999815; 23) = 4.17$ .

If  $|t_{ij}| \leq 4.17$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

18.7. a. See Problem 16.10c.

b.  $r = .984$

d.  $t_{ij}$ :

$i$	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$
1	.9927	2.4931	-.3265	.3265	-.3265	.3265
2	.1630	-.4907	-.4907	.8234	-1.1646	.8234
3	1.0497	-.9360	2.5645	-.2719	.3811	1.0497

  

$i$	$j = 7$	$j = 8$	$j = 9$	$j = 10$	$j = 11$	$j = 12$
1	-.9927	.9927	-1.7017	.3265	-1.7017	-.3265
2	-.4907	1.5185	.1630	-.4907	-1.1646	.8234
3	-.2719	-.9360	-1.6401	-.9360	.3811	-.2719

$H_0$ : no outliers,  $H_a$ : at least one outlier.  $t(.99965; 32) = 3.75$ .

If  $|t_{ij}| \leq 3.75$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

18.8. a. See Problem 16.11c.



b.  $r = .992$

d.  $t_{ij}$ :

$i$	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$	$j = 7$
1	-1.2477	.7360	-.0203	.6192	1.8045	.1538	-.6601
2	1.5815	-.4677	-.4095	1.6415	.2874	-.7594	-1.8287
3	-1.4648	1.8864	-.8150	-.0580	-1.4052	-.6396	.4648
4	.7243	1.2537	.9000	-.4386	-.5551	1.0764	.2003
5	-1.8560	.8443	-.3775	-.0871	.6105	.1451	-.6688
6	-.5901	-1.1767	.7477	1.8773	-.1829	.6893	-.4153
$i$	$j = 8$	$j = 9$	$j = 10$	$j = 11$	$j = 12$	$j = 13$	$j = 14$
1	-2.0298	1.1472	-1.6656	1.8651	-.8355	-.5434	1.2063
2	.8121	-.7594	-1.2892	-.8179	2.0053	1.3427	.5784
3	-1.2863	.0580	.9323	.0580	.5230	.7565	1.4648
4	-1.3189	.6659	-.1480	-2.1035	-.2061	-1.0823	-1.3784
5	.5522	.9616	.0871	.4357	1.0204	-1.3754	.8443
6	.1074	1.6355	-1.2952	.2816	-.8238	-.2990	.0493
$i$	$j = 15$	$j = 16$	$j = 17$	$j = 18$	$j = 19$	$j = 20$	
1	.0958	.3281	1.0882	-.0203	-.4852	-1.5455	
2	.9881	-.8765	-.1190	-.9940	-.4677	-.4095	
3	-.0580	.1161	-.1161	.2322	-1.5246	.8736	
4	.6659	1.0175	.8414	.3165	.4910	-.9646	
5	-1.9168	.6688	.4357	.0871	1.3160	-1.7954	
6	-1.2359	-.1248	1.6355	1.1590	-1.4141	-.5901	

$H_0$  : no outliers,  $H_a$  : at least one outlier.  $t(.9999417; 113) = 4.08$ .

If  $|t_{ij}| \leq 4.08$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

18.11.  $H_0$ : all  $\sigma_i^2$  are equal ( $i = 1, 2, 3$ ),  $H_a$ : not all  $\sigma_i^2$  are equal.

$\tilde{Y}_1 = 6.80$ ,  $\tilde{Y}_2 = 8.20$ ,  $\tilde{Y}_3 = 9.55$ ,  $MSTR = .0064815$ ,  $MSE = .26465$ ,

$F_L^* = .0064815/.26465 = .02$ ,  $F(.95; 2, 24) = 3.40$ . If  $F_L^* \leq 3.40$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .98

18.13. a.  $H_0$ : all  $\sigma_i^2$  are equal ( $i = 1, 2, 3$ ),  $H_a$ : not all  $\sigma_i^2$  are equal.

$s_1 = 1.7321$ ,  $s_2 = 1.2881$ ,  $s_3 = 1.6765$ ,  $n_i \equiv 12$ ,  $H^* = (1.7321)^2/(1.2881)^2 = 1.808$ ,  $H(.99; 3, 11) = 6.75$ . If  $H^* \leq 6.75$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value  $> .05$

b.  $\tilde{Y}_1 = 21.5$ ,  $\tilde{Y}_2 = 27.5$ ,  $\tilde{Y}_3 = 21.0$ ,  $MSTR = .19444$ ,  $MSE = .93434$ ,

$F_L^* = .19444/.93434 = .21$ ,  $F(.99; 2, 33) = 5.31$ . If  $F_L^* \leq 5.31$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .81

18.14. a.  $H_0$ : all  $\sigma_i^2$  are equal ( $i = 1, \dots, 6$ ),  $H_a$ : not all  $\sigma_i^2$  are equal.

$s_1 = .1925$ ,  $s_2 = .1854$ ,  $s_3 = .1646$ ,  $s_4 = .1654$ ,  $s_5 = .1727$ ,  $s_6 = .1735$ ,  $n_i \equiv 20$ ,  $H^* = (.1925)^2/ (.1646)^2 = 1.3677$ ,  $H(.99; 6, 19) = 5.2$ . If  $H^* \leq 5.2$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value  $> .05$

- b.  $\tilde{Y}_1 = .08, \tilde{Y}_2 = .12, \tilde{Y}_3 = .47, \tilde{Y}_4 = .41, \tilde{Y}_5 = .175, \tilde{Y}_6 = .125, MSTR = .002336, MSE = .012336, F_L^* = .002336/.012336 = .19, F(.99; 5, 114) = 3.18$ . If  $F_L^* \leq 3.18$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .97

18.17. a.  $\bar{Y}_1 = 3.5625, \bar{Y}_2 = 5.8750, \bar{Y}_3 = 10.6875, \bar{Y}_4 = 15.5625$

$e_{ij}$ :

$i$	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$
1	.4375	-.5625	-1.5625	-.5625	.4375	.4375
2	1.1250	.1250	-1.8750	.1250	1.1250	-3.8750
3	1.3125	-4.6875	3.3125	1.3125	-.6875	-1.6875
4	.4375	-1.5625	-9.5625	3.4375	-3.5625	-5.5625

$i$	$j = 7$	$j = 8$	$j = 9$	$j = 10$	$j = 11$	$j = 12$
1	-.5625	2.4375	1.4375	.4375	-1.5625	.4375
2	3.1250	-.8750	-.8750	3.1250	-2.8750	2.1250
3	1.3125	6.3125	-3.6875	-4.6875	1.3125	.3125
4	-.5625	8.4375	-5.5625	7.4375	1.4375	4.4375

$i$	$j = 13$	$j = 14$	$j = 15$	$j = 16$
1	.4375	-1.5625	-.5625	.4375
2	.1250	-1.8750	1.1250	.1250
3	-4.6875	2.3125	-.6875	3.3125
4	-.5625	2.4375	-7.5625	6.4375

- c.  $H_0$ : all  $\sigma_i^2$  are equal ( $i = 1, 2, 3, 4$ ),  $H_a$ : not all  $\sigma_i^2$  are equal.

$\tilde{Y}_1 = 4.0, \tilde{Y}_2 = 6.0, \tilde{Y}_3 = 11.5, \tilde{Y}_4 = 16.5, MSTR = 37.1823, MSE = 3.8969, F_L^* = 37.1823/3.8969 = 9.54, F(.95; 3, 60) = 2.76$ . If  $F_L^* \leq 2.76$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

d.

$i$	$\bar{Y}_i$	$s_i$
1	3.5625	1.0935
2	5.8750	1.9958
3	10.6875	3.2397
4	16.5625	5.3786

e.

$\lambda$	$SSE$	$\lambda$	$SSE$
-1.0	1,038.26	.1	410.65
-.8	790.43	.2	410.92
-.6	624.41	.4	430.49
-.4	516.16	.6	476.68
-.2	450.16	.8	553.64
-.1	429.84	1.0	669.06
0	416.84		

Yes

18.18. a.  $\bar{Y}'_1 = .5314, \bar{Y}'_2 = .7400, \bar{Y}'_3 = 1.0080, \bar{Y}'_4 = 1.1943$

$e'_{ij}$ :

$i$	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$	$j = 7$	$j = 8$
1	.071	-.054	-.230	-.054	.071	.071	-.054	.247
2	.105	.038	-.138	.038	.105	-.439	.214	-.041
3	.071	-.230	.138	.071	-.008	-.054	.071	.222
4	.036	-.018	-.349	.107	-.080	-.153	.010	.204

  

$i$	$j = 9$	$j = 10$	$j = 11$	$j = 12$	$j = 13$	$j = 14$	$j = 15$	$j = 16$
1	.168	.071	-.230	.071	.071	-.230	-.054	.071
2	-.041	.214	-.263	.163	.038	-.138	.105	.038
3	-.163	-.230	.071	.033	-.230	.106	-.008	.138
4	-.153	.186	.061	.128	.010	.085	-.240	.167

b.  $r = .971$

c.  $H_0$ : all  $\sigma_i^2$  are equal ( $i = 1, 2, 3, 4$ ),  $H_a$ : not all  $\sigma_i^2$  are equal.

$\tilde{Y}_1 = .6021$ ,  $\tilde{Y}_2 = .7782$ ,  $\tilde{Y}_3 = 1.0603$ ,  $\tilde{Y}_4 = 1.2173$ ,  $MSTR = .001214$ ,

$MSE = .01241$ ,  $F_L^* = .001214/.01241 = .10$ ,  $F(.95; 3, 60) = 2.76$ .

If  $F_L^* \leq 2.76$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

18.20.

$i$ :	1	2	3	4
$s_i$ :	1.09354	1.99583	3.23973	5.37858
$w_i$ :	.83624	.25105	.09528	.034567

$H_0$ : all  $\mu_i$  are equal ( $i = 1, 2, 3, 4$ ),  $H_a$ : not all  $\mu_i$  are equal.

$SSE_w(F) = 60$ ,  $df_F = 60$ ,  $SSE_w(R) = 213.9541$ ,  $df_R = 63$ ,

$F_w^* = [(213.9541 - 60)/3] \div (60/60) = 51.32$ ,  $F(.99; 3, 60) = 4.13$ .

If  $F_w^* \leq 4.13$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .

18.23. a.  $H_0$ : all  $\mu_i$  are equal ( $i = 1, 2, 3, 4$ ),  $H_a$ : not all  $\mu_i$  are equal.

$MSTR = 470.8125$ ,  $MSE = 28.9740$ ,  $F_R^* = 470.8125/28.9740 = 16.25$ ,

$F(.95; 2, 24) = 3.40$ . If  $F_R^* \leq 3.40$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .

b.  $P$ -value = 0+

e.  $\bar{R}_1 = 6.50$ ,  $\bar{R}_2 = 15.50$ ,  $\bar{R}_3 = 22.25$ ,  $B = z(.9833) = 2.13$

Comparison	Testing Limits	
1 and 2	$-9.00 \pm 2.13(3.500)$	$-16.455$ and $-1.545$
1 and 3	$15.75 \pm 2.13(4.183)$	$-24.660$ and $-9.840$
2 and 3	$-6.75 \pm 2.13(3.969)$	$-15.204$ and $1.704$

  

Group 1	Group 2
Low Level $i = 1$	Moderate level $i = 2$
	High level $i = 3$

18.24. a.  $H_0$ : all  $\mu_i$  are equal ( $i = 1, 2, 3$ ),  $H_a$ : not all  $\mu_i$  are equal.

$MSTR = 1,297.0000$ ,  $MSE = 37.6667$ ,  $F_R^* = 1,297.0000/37.6667 = 34.43$ ,

$F(.99; 2, 33) = 5.31$ . If  $F_R^* \leq 5.31$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .

b.  $P$ -value = 0+

e.  $\bar{R}_1 = 12.792$ ,  $\bar{R}_2 = 30.500$ ,  $\bar{R}_3 = 12.208$ ,  $B = z(.9833) = 2.128$

Comparison	Testing Limits							
1 and 2	$-17.708 \pm 2.128(4.301)$	-26.861 and -8.555						
1 and 3	$.584 \pm 2.128(4.301)$	-8.569 and 9.737						
2 and 3	$18.292 \pm 2.128(4.301)$	9.140 and 27.445						
<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: center; border-bottom: 1px solid black; width: 50%;"><b>Group 1</b></th> <th style="text-align: center; border-bottom: 1px solid black; width: 50%;"><b>Group 2</b></th> </tr> </thead> <tbody> <tr> <td style="padding-left: 20px;">Young <math>i = 1</math></td> <td style="padding-left: 20px;">Middle <math>i = 2</math></td> </tr> <tr> <td style="padding-left: 20px;">Elderly <math>i = 3</math></td> <td></td> </tr> </tbody> </table>			<b>Group 1</b>	<b>Group 2</b>	Young $i = 1$	Middle $i = 2$	Elderly $i = 3$	
<b>Group 1</b>	<b>Group 2</b>							
Young $i = 1$	Middle $i = 2$							
Elderly $i = 3$								



# Chapter 19

## TWO-FACTOR STUDIES WITH EQUAL SAMPLE SIZES

- 19.4. a.  $\mu_1 = 31, \mu_2 = 37$   
 b.  $\alpha_1 = \mu_1 - \mu_{..} = 31 - 34 = -3, \alpha_2 = \mu_2 - \mu_{..} = 37 - 34 = 3$

- 19.7. a.  $E\{MSE\} = 1.96, E\{MSA\} = 541.96$

- 19.10. a.  $\bar{Y}_{11.} = 21.66667, \bar{Y}_{12.} = 21.33333, \bar{Y}_{21.} = 27.83333,$   
 $\bar{Y}_{22.} = 27.66667, \bar{Y}_{31.} = 22.33333, \bar{Y}_{32.} = 20.50000$

- b.  $e_{ijk}$ :

$i$	$j = 1$	$j = 2$	$i$	$j = 1$	$j = 2$	$i$	$j = 1$	$j = 2$
1	-.66667	-.33333	2	2.16667	-1.66667	3	2.66667	2.50000
	1.33333	.66667		1.16667	1.33333		-.33333	-1.50000
	-2.66667	-1.33333		-1.83333	-.66667		.66667	-.50000
	.33333	-.33333		.16667	.33333		-1.33333	.50000
	.33333	-2.33333		-.83333	-.66667		-.33333	-.50000
	1.33333	3.66667		-.83333	1.33333		-1.33333	-.50000

- d.  $r = .986$

- 19.11. b.

Source	$SS$	$df$	$MS$
Treatments	327.222	5	65.444
A (age)	316.722	2	158.361
B (gender)	5.444	1	5.444
AB interactions	5.056	2	2.528
Error	71.667	30	2.389
Total	398.889	35	

Yes, factor  $A$  (age) accounts for most of the total variability.

- c.  $H_0$ : all  $(\alpha\beta)_{ij}$  equal zero,  $H_a$ : not all  $(\alpha\beta)_{ij}$  equal zero.

$$F^* = 2.528/2.389 = 1.06, F(.95; 2, 30) = 3.32.$$

If  $F^* \leq 3.32$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .36

d.  $H_0$ : all  $\alpha_i$  equal zero ( $i = 1, 2, 3$ ),  $H_a$ : not all  $\alpha_i$  equal zero.

$$F^* = 158.361/2.389 = 66.29, F(.95; 2, 30) = 3.32.$$

If  $F^* \leq 3.32$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

$H_0$ :  $\beta_1 = \beta_2 = 0$ ,  $H_a$ : not both  $\beta_1$  and  $\beta_2$  equal zero.

$$F^* = 5.444/2.389 = 2.28, F(.95; 1, 30) = 4.17.$$

If  $F^* \leq 4.17$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .14

e.  $\alpha \leq .143$

g.  $SSA = SSTR, SSB + SSAB + SSE = SSE$ , yes

19.14. a.  $\bar{Y}_{11.} = 2.475, \bar{Y}_{12.} = 4.600, \bar{Y}_{13.} = 4.575, \bar{Y}_{21.} = 5.450, \bar{Y}_{22.} = 8.925,$   
 $\bar{Y}_{23.} = 9.125, \bar{Y}_{31.} = 5.975, \bar{Y}_{32.} = 10.275, \bar{Y}_{33.} = 13.250$

b.  $e_{ijk}$ :

$i$	$j = 1$	$j = 2$	$j = 3$	$i$	$j = 1$	$j = 2$	$j = 3$
1	-.075	0	.225	2	.350	-.025	-.025
	.225	-.400	-.075		-.250	.175	.175
	-.175	.300	-.175		.050	-.225	-.425
	.025	.100	.025		-.150	.075	.275
$i$	$j = 1$	$j = 2$	$j = 3$				
3	.125	-.375	.250				
	-.275	.225	-.250				
	-.075	.325	.050				
	.225	-.175	-.050				

d.  $r = .988$

19.15. b.

Source	$SS$	$df$	$MS$
Treatments	373.105	8	46.638
A (ingredient 1)	220.020	2	110.010
B (ingredient 2)	123.660	2	61.830
AB interactions	29.425	4	7.356
Error	1.625	27	.0602
Total	374.730	35	

c.  $H_0$ : all  $(\alpha\beta)_{ij}$  equal zero,  $H_a$ : not all  $(\alpha\beta)_{ij}$  equal zero.  $F^* = 7.356/.0602 = 122.19, F(.95; 4, 27) = 2.73$ . If  $F^* \leq 2.73$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

d.  $H_0$ : all  $\alpha_i$  equal zero ( $i = 1, 2, 3$ ),  $H_a$ : not all  $\alpha_i$  equal zero.  $F^* = 110.010/.0602 = 1,827.41, F(.95; 2, 27) = 3.35$ . If  $F^* \leq 3.35$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

$H_0$ : all  $\beta_j$  equal zero ( $j = 1, 2, 3$ ),  $H_a$ : not all  $\beta_j$  equal zero.  $F^* = 61.830/.0602 = 1,027.08, F(.95; 2, 27) = 3.35$ . If  $F^* \leq 3.35$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

e.  $\alpha \leq .143$

19.20. a.  $\bar{Y}_{11.} = 222.00, \bar{Y}_{12.} = 106.50, \bar{Y}_{13.} = 60.50, \bar{Y}_{21.} = 62.25, \bar{Y}_{22.} = 44.75, \bar{Y}_{23.} = 38.75$

b.  $e_{ijk}$ :

$i$	$j = 1$	$j = 2$	$j = 3$	$i$	$j = 1$	$j = 2$	$j = 3$
1	18	3.5	-4.5	2	8.75	2.25	-1.75
	-16	11.5	-.5		-9.25	7.25	-5.75
	-5	-3.5	7.5		5.75	-13.75	1.25
	3	-11.5	-2.5		-5.25	4.25	6.25

d.  $r = .994$

19.21. b.

Source	$SS$	$df$	$MS$
Treatments	96,024.37500	5	19,204.87500
$A$ (type)	39,447.04167	1	39,447.04167
$B$ (years)	36,412.00000	2	18,206.00000
$AB$ interactions	20,165.33333	2	10,082.66667
Error	1,550.25000	18	86.12500
Total	97,574.62500	23	

c.  $H_0$ : all  $(\alpha\beta)_{ij}$  equal zero,  $H_a$ : not all  $(\alpha\beta)_{ij}$  equal zero.  $F^* = 10,082.66667/86.12500 = 117.07, F(.99; 2, 18) = 6.01$ . If  $F^* \leq 6.01$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

d.  $H_0$ :  $\alpha_1 = \alpha_2 = 0, H_a$ : not both  $\alpha_1$  and  $\alpha_2$  equal zero.  $F^* = 39,447.04167/86.12500 = 458.02, F(.99; 1, 18) = 8.29$ . If  $F^* \leq 8.29$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

$H_0$ : all  $\beta_j$  equal zero ( $j = 1, 2, 3$ ),  $H_a$ : not all  $\beta_j$  equal zero.  $F^* = 18,206.00000/86.12500 = 211.39, F(.99; 2, 18) = 6.01$ . If  $F^* \leq 6.01$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

e.  $\alpha \leq .030$

19.27. a.  $B = t(.9975; 75) = 2.8925, q(.95; 5, 75) = 3.96, T = 2.800$

b.  $B = t(.99167; 27) = 2.552, q(.95; 3, 27) = 3.51, T = 2.482$

19.30. a.  $s\{\bar{Y}_{11.}\} = .631, t(.975; 30) = 2.042, 21.66667 \pm 2.042(.631), 20.378 \leq \mu_{11} \leq 22.955$

b.  $\bar{Y}_{1.} = 23.94, \bar{Y}_{2.} = 23.17$

c.  $\hat{D} = .77, s\{\hat{D}\} = .5152, t(.975; 30) = 2.042, .77 \pm 2.042(.5152), -.282 \leq D \leq 1.822$

d.  $\bar{Y}_{1..} = 21.50, \bar{Y}_{2..} = 27.75, \bar{Y}_{3..} = 21.42$

e.  $\hat{D}_1 = \bar{Y}_{1..} - \bar{Y}_{2..} = -6.25, \hat{D}_2 = \bar{Y}_{1..} - \bar{Y}_{3..} = .08, \hat{D}_3 = \bar{Y}_{2..} - \bar{Y}_{3..} = 6.33, s\{\hat{D}_i\} = .631$   
 $(i = 1, 2, 3), q(.90; 3, 30) = 3.02, T = 2.1355$

$$-6.25 \pm 2.1355(.631) \quad -7.598 \leq D_1 \leq -4.902$$

$$.08 \pm 2.1355(.631) \quad -1.268 \leq D_2 \leq 1.428$$

$$6.33 \pm 2.1355(.631) \quad 4.982 \leq D_3 \leq 7.678$$



- f. Yes
- g.  $\hat{L} = -6.29$ ,  $s\{\hat{L}\} = .5465$ ,  $t(.976; 30) = 2.042$ ,  $-6.29 \pm 2.042(.5465)$ ,  $-7.406 \leq L \leq -5.174$
- h.  $L = .3\mu_{12} + .6\mu_{22} + .1\mu_{32}$ ,  $\hat{L} = 25.05000$ ,  $s\{\hat{L}\} = .4280$ ,  $t(.975; 30) = 2.042$ ,  $25.05000 \pm 2.042(.4280)$ ,  $24.176 \leq L \leq 25.924$

- 19.32. a.  $s\{\bar{Y}_{23}\} = .1227$ ,  $t(.975; 27) = 2.052$ ,  $9.125 \pm 2.052(.1227)$ ,  $8.873 \leq \mu_{23} \leq 9.377$
- b.  $\hat{D} = 2.125$ ,  $s\{\hat{D}\} = .1735$ ,  $t(.975; 27) = 2.052$ ,  $2.125 \pm 2.052(.1735)$ ,  $1.769 \leq D \leq 2.481$
- c.  $\hat{L}_1 = 2.1125$ ,  $\hat{L}_2 = 3.5750$ ,  $\hat{L}_3 = 5.7875$ ,  $\hat{L}_4 = 1.4625$ ,  $\hat{L}_5 = 3.6750$ ,  $\hat{L}_6 = 2.2125$ ,  $s\{\hat{L}_i\} = .1502$  ( $i = 1, 2, 3$ ),  $s\{\hat{L}_i\} = .2125$  ( $i = 4, 5, 6$ ),  $F(.90; 8, 27) = 1.90$ ,  $S = 3.899$

$$\begin{array}{ll} 2.1125 \pm 3.899(.1502) & 1.527 \leq L_1 \leq 2.698 \\ 3.5750 \pm 3.899(.1502) & 2.989 \leq L_2 \leq 4.161 \\ 5.7875 \pm 3.899(.1502) & 5.202 \leq L_3 \leq 6.373 \\ 1.4625 \pm 3.899(.2125) & .634 \leq L_4 \leq 2.291 \\ 3.6750 \pm 3.899(.2125) & 2.846 \leq L_5 \leq 4.504 \\ 2.2125 \pm 3.899(.2125) & 1.384 \leq L_6 \leq 3.041 \end{array}$$

- d.  $s\{\hat{D}_i\} = .1735$ ,  $q(.90; 9, 27) = 4.31$ ,  $T = 3.048$ ,  $Ts\{\hat{D}_i\} = .529$ ,  $\bar{Y}_{33} = 13.250$

e.

$i$	$j$	$1/\bar{Y}_{ij}$	$\sqrt{\bar{Y}_{ij}}$
1	1	.404	1.573
1	2	.217	2.145
1	3	.219	2.139
2	1	.183	2.335
2	2	.112	2.987
2	3	.110	3.021
3	1	.167	2.444
3	2	.097	3.205
3	3	.075	3.640

- 19.35. a.  $s\{\bar{Y}_{23}\} = 4.6402$ ,  $t(.995; 18) = 2.878$ ,  $38.75 \pm 2.878(4.6402)$ ,  $25.3955 \leq \mu_{23} \leq 52.1045$
- b.  $\hat{D} = 46.00$ ,  $s\{\hat{D}\} = 6.5622$ ,  $t(.995; 18) = 2.878$ ,  $46.00 \pm 2.878(6.5622)$ ,  $27.114 \leq D \leq 64.886$
- c.  $F(.95; 5, 18) = 2.77$ ,  $S = 3.7216$ ,  $B = t(.99583; 18) = 2.963$
- d.  $\hat{D}_1 = 159.75$ ,  $\hat{D}_2 = 61.75$ ,  $\hat{D}_3 = 21.75$ ,  $\hat{L}_1 = 98.00$ ,  $\hat{L}_2 = 138.00$ ,  $\hat{L}_3 = 40.00$ ,  $s\{\hat{D}_i\} = 6.5622$  ( $i = 1, 2, 3$ ),  $s\{\hat{L}_i\} = 9.2804$  ( $i = 1, 2, 3$ ),  $B = t(.99583; 18) = 2.963$

$159.75 \pm 2.963(6.5622)$	$140.31 \leq D_1 \leq 179.19$
$61.75 \pm 2.963(6.5622)$	$42.31 \leq D_2 \leq 81.19$
$21.75 \pm 2.963(6.5622)$	$2.31 \leq D_3 \leq 41.19$
$98.00 \pm 2.963(9.2804)$	$70.50 \leq L_1 \leq 125.50$
$138.00 \pm 2.963(9.2804)$	$110.50 \leq L_2 \leq 165.50$
$40.00 \pm 2.963(9.2804)$	$12.50 \leq L_3 \leq 67.50$

e.  $q(.95; 6, 18) = 4.49$ ,  $T = 3.1749$ ,  $s\{\hat{D}\} = 6.5622$ ,  $Ts\{\hat{D}\} = 20.834$ ,  $\bar{Y}_{23} = 38.75$ ,  
 $\bar{Y}_{22} = 44.75$

f.  $B = t(.9875; 18) = 2.445$ ,  $s\{\bar{Y}_{ij}\} = 4.6402$

$44.75 \pm 2.445(4.6402)$	$33.405 \leq \mu_{22} \leq 56.095$
$38.75 \pm 2.445(4.6402)$	$27.405 \leq \mu_{23} \leq 50.095$

g.

$i$	$j$	$1/\bar{Y}_{ij}$	$\log_{10}\bar{Y}_{ij}$
1	1	.00450	2.346
1	2	.00939	2.027
1	3	.01653	1.782
2	1	.01606	1.794
2	2	.02235	1.651
2	3	.02581	1.588

19.38.  $\Delta/\sigma = 2$ ,  $2n = 8$ ,  $n = 4$

19.40.  $.5\sqrt{n}/.29 = 4.1999$ ,  $n = 6$

19.42.  $8\sqrt{n}/9.1 = 3.1591$ ,  $n = 13$



# Chapter 20

## TWO-FACTOR STUDIES – ONE CASE PER TREATMENT

20.2. b.

Source	<i>SS</i>	<i>df</i>	<i>MS</i>
Location	37.0050	3	12.3350
Week	47.0450	1	47.0450
Error	.3450	3	.1150
Total	84.3950	7	

$H_0$ : all  $\alpha_i$  equal zero ( $i = 1, \dots, 4$ ),  $H_a$ : not all  $\alpha_i$  equal zero.

$F^* = 12.3350/.1150 = 107.26$ ,  $F(.95; 3, 3) = 9.28$ . If  $F^* \leq 9.28$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = .0015

$H_0$ :  $\beta_1 = \beta_2 = 0$ ,  $H_a$ : not both  $\beta_1$  and  $\beta_2$  equal zero.

$F^* = 47.0450/.1150 = 409.09$ ,  $F(.95; 1, 3) = 10.1$ . If  $F^* \leq 10.1$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = .0003.  $\alpha \leq .0975$

c.  $\hat{D}_1 = \bar{Y}_1 - \bar{Y}_2 = 18.95 - 14.55 = 4.40$ ,  $\hat{D}_2 = \bar{Y}_1 - \bar{Y}_3 = 18.95 - 14.60 = 4.35$ ,  
 $\hat{D}_3 = \bar{Y}_1 - \bar{Y}_4 = 18.95 - 18.80 = .15$ ,  $\hat{D}_4 = \bar{Y}_2 - \bar{Y}_3 = -.05$ ,  $\hat{D}_5 = \bar{Y}_2 - \bar{Y}_4 = -4.25$ ,  
 $\hat{D}_6 = \bar{Y}_3 - \bar{Y}_4 = -4.20$ ,  $\hat{D}_7 = \bar{Y}_1 - \bar{Y}_2 = 14.30 - 19.15 = -4.85$ ,  $s\{\hat{D}_i\} = .3391$   
( $i = 1, \dots, 6$ ),  $s\{\hat{D}_7\} = .2398$ ,  $B = t(.99286; 3) = 5.139$

$$4.40 \pm 5.139(.3391) \quad 2.66 \leq D_1 \leq 6.14$$

$$4.35 \pm 5.139(.3391) \quad 2.61 \leq D_2 \leq 6.09$$

$$.15 \pm 5.139(.3391) \quad -1.59 \leq D_3 \leq 1.89$$

$$-.05 \pm 5.139(.3391) \quad -1.79 \leq D_4 \leq 1.69$$

$$-4.25 \pm 5.139(.3391) \quad -5.99 \leq D_5 \leq -2.51$$

$$-4.20 \pm 5.139(.3391) \quad -5.94 \leq D_6 \leq -2.46$$

$$-4.85 \pm 5.139(.2398) \quad -6.08 \leq D_7 \leq -3.62$$

20.3. a.  $\hat{\mu}_{32} = \bar{Y}_3 + \bar{Y}_2 - \bar{Y}_.. = 14.600 + 19.150 - 16.725 = 17.025$

b.  $s^2\{\hat{\mu}_{32}\} = .071875$

c.  $s\{\hat{\mu}_{32}\} = .2681$ ,  $t(.975; 3) = 3.182$ ,  $17.025 \pm 3.182(.2681)$ ,  $16.172 \leq \mu_{32} \leq 17.878$

21.4.  $\hat{D} = (-4.13473)/(18.5025)(11.76125) = -.019$ ,  $SSAB^* = .0786$ ,  $SSRem^* = .2664$ .

$H_0: D = 0, H_a: D \neq 0. F^* = (.0786/1) \div (.2664/2) = .59, F(.975; 1, 2) = 38.5.$   
If  $F^* \leq 38.5$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

# Chapter 21

## RANDOMIZED COMPLETE BLOCK DESIGNS

21.5. b.  $e_{ij}$ :

$i$	$j = 1$	$j = 2$	$j = 3$
1	-2.50000	1.50000	1.00000
2	1.50000	-.50000	-1.00000
3	2.16667	-.83333	-1.33333
4	.16667	-.83333	.66667
5	4.16667	-4.83333	.66667
6	1.50000	-.50000	-1.00000
7	-1.50000	-1.50000	3.00000
8	-2.83333	3.16667	-.33333
9	-1.50000	2.50000	-1.00000
10	-1.16667	1.83333	-.66667

$r = .984$

- d.  $H_0: D = 0, H_a: D \neq 0$ .  $SSBLTR^* = .13, SSRem^* = 112.20$ ,  
 $F^* = (.13/1) \div (112.20/17) = .02, F(.99; 1, 17) = 8.40$ . If  $F^* \leq 8.40$  conclude  $H_0$ ,  
 otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .89

21.6. a.

Source	$SS$	$df$	$MS$
Blocks	433.36667	9	48.15185
Training methods	1,295.00000	2	647.50000
Error	112.33333	18	6.24074
Total	1,840.70000	29	

- b.  $\bar{Y}_1 = 70.6, \bar{Y}_2 = 74.6, \bar{Y}_3 = 86.1$
- c.  $H_0$ : all  $\tau_j$  equal zero ( $j = 1, 2, 3$ ),  $H_a$ : not all  $\tau_j$  equal zero.  
 $F^* = 647.50000/6.24074 = 103.754, F(.95; 2, 18) = 3.55$ . If  $F^* \leq 3.55$  conclude  
 $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+
- d.  $\hat{D}_1 = \bar{Y}_1 - \bar{Y}_2 = -4.0, \hat{D}_2 = \bar{Y}_1 - \bar{Y}_3 = -15.5, \hat{D}_3 = \bar{Y}_2 - \bar{Y}_3 = -11.5$ ,  
 $s\{\hat{D}_i\} = 1.1172$  ( $i = 1, 2, 3$ ),  $q(.90; 3, 18) = 3.10, T = 2.192$

$$\begin{array}{ll}
-4.0 \pm 2.192(1.1172) & -6.45 \leq D_1 \leq -1.55 \\
-15.5 \pm 2.192(1.1172) & -17.95 \leq D_2 \leq -13.05 \\
-11.5 \pm 2.192(1.1172) & -13.95 \leq D_3 \leq -9.05
\end{array}$$

e.  $H_0$ : all  $\rho_i$  equal zero ( $i = 1, \dots, 10$ ),  $H_a$ : not all  $\rho_i$  equal zero.

$$F^* = 48.15185/6.24074 = 7.716, F(.95; 9, 18) = 2.46.$$

If  $F^* \leq 2.46$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = .0001

21.12. b.  $\bar{Y}_{1..} = 7.25, \bar{Y}_{2..} = 12.75, \hat{L} = \bar{Y}_{1..} - \bar{Y}_{2..} = -5.50, s\{\hat{L}\} = 1.25,$   
 $t(.995; 8) = 3.355, -5.50 \pm 3.355(1.25), -9.69 \leq L \leq -1.31$

21.14.  $\phi = \frac{1}{2.5} \sqrt{\frac{10(18)}{3}} = 3.098, \nu_1 = 2, \nu_2 = 27, 1 - \beta > .99$

21.16.  $n = 49$  blocks

21.18.  $\hat{E} = 3.084$

# Chapter 22

## ANALYSIS OF COVARIANCE

22.7. a.  $e_{ij}$  :

$i$	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$
1	-.5281	.4061	.0089	.4573	-.1140	-.1911
2	-.2635	-.2005	.3196	.2995	-.1662	.0680
3	-.1615	.2586	-.0099	-.3044	.0472	.1700

  

$i$	$j = 7$	$j = 8$	$j = 9$	$j = 10$	$j = 11$	$j = 12$
1	.0660	-.0939	-.0112			
2	-.0690	-.1776	-.0005	.0653	.0251	.0995

b.  $r = .988$

c.  $Y_{ij} = \mu. + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \gamma x_{ij} + \beta_1 I_{ij1} x_{ij} + \beta_2 I_{ij2} x_{ij} + \varepsilon_{ij}$

$H_0: \beta_1 = \beta_2 = 0$ ,  $H_a$ : not both  $\beta_1$  and  $\beta_2$  equal zero.

$SSE(F) = .9572$ ,  $SSE(R) = 1.3175$ ,

$F^* = (.3603/2) \div (.9572/21) = 3.95$ ,  $F(.99; 2, 21) = 5.78$ .

If  $F^* \leq 5.78$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .035

d. Yes, 5

22.8. b. Full model:  $Y_{ij} = \mu. + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \gamma x_{ij} + \varepsilon_{ij}$ , ( $\bar{X}_{..} = 9.4$ ).

Reduced model:  $Y_{ij} = \mu. + \gamma x_{ij} + \varepsilon_{ij}$ .

c. Full model:  $\hat{Y} = 7.80627 + 1.65885I_1 - .17431I_2 + 1.11417x$ ,  $SSE(F) = 1.3175$

Reduced model:  $\hat{Y} = 7.95185 + .54124x$ ,  $SSE(R) = 5.5134$

$H_0: \tau_1 = \tau_2 = 0$ ,  $H_a$ : not both  $\tau_1$  and  $\tau_2$  equal zero.

$F^* = (4.1959/2) \div (1.3175/23) = 36.625$ ,  $F(.95; 2, 23) = 3.42$ .

If  $F^* \leq 3.42$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

d.  $MSE(F) = .0573$ ,  $MSE = .6401$

e.  $\hat{Y} = \hat{\mu}_. + \hat{\tau}_2 - .4\hat{\gamma} = 7.18629$ ,  $s^2\{\hat{\mu}_.\} = .00258$ ,  $s^2\{\hat{\tau}_2\} = .00412$ ,  $s^2\{\hat{\gamma}\} = .00506$ ,  
 $s\{\hat{\mu}_., \hat{\tau}_2\} = -.00045$ ,  $s\{\hat{\tau}_2, \hat{\gamma}\} = -.00108$ ,  $s\{\hat{\mu}_., \hat{\gamma}\} = -.00120$ ,  $s\{\hat{Y}\} = .09183$ ,  
 $t(.975; 23) = 2.069$ ,  $7.18629 + 2.069(.09183)$ ,  $6.996 \leq \mu. + \tau_2 - .4\gamma \leq 7.376$



- f.  $\hat{D}_1 = \hat{\tau}_1 - \hat{\tau}_2 = 1.83316$ ,  $\hat{D}_2 = \hat{\tau}_1 - \hat{\tau}_3 = 2\hat{\tau}_1 + \hat{\tau}_2 = 3.14339$ ,  $\hat{D}_3 = \hat{\tau}_2 - \hat{\tau}_3 = 2\hat{\tau}_2 + \hat{\tau}_1 = 1.31023$ ,  $s^2\{\hat{\tau}_1\} = .03759$ ,  $s\{\hat{\tau}_1, \hat{\tau}_2\} = -.00418$ ,  $s\{\hat{D}_1\} = .22376$ ,  $s\{\hat{D}_2\} = .37116$ ,  $s\{\hat{D}_3\} = .19326$ ,  $F(.90; 2, 23) = 2.55$ ,  $S = 2.258$
- $$1.83316 \pm 2.258(.22376) \quad 1.328 \leq D_1 \leq 2.338$$
- $$3.14339 \pm 2.258(.37116) \quad 2.305 \leq D_2 \leq 3.981$$
- $$1.31023 \pm 2.258(.19326) \quad .874 \leq D_3 \leq 1.747$$

22.15. a.  $e_{ijk}$ :

$i$	$j = 1$	$j = 2$	$i$	$j = 1$	$j = 2$	$i$	$j = 1$	$j = 2$
1	-.1184	-.3510	2	-.6809	.2082	3	.9687	.6606
	-.3469	-.0939		.8660	-.1877		-.0150	.0565
	.0041	.0286		-.1177	.2531		.8789	-.1109
	-.6041	.0735		.2905	-.2327		-1.1211	-.0660
	1.2000	-.0163		-.3912	-.2776		.0912	-.4293
	-.1347	.3592		.0333	.2367		-.8027	-.1109

b.  $r = .974$

c. 
$$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \alpha_2 I_{ijk2} + \beta_1 I_{ijk3} + (\alpha\beta)_{11} I_{ijk1} I_{ijk3}$$

$$+ (\alpha\beta)_{21} I_{ijk2} I_{ijk3} + \gamma x_{ijk} + \delta_1 I_{ijk1} x_{ijk} + \delta_2 I_{ijk2} x_{ijk}$$

$$+ \delta_3 I_{ijk3} x_{ijk} + \delta_4 I_{ijk1} I_{ijk3} x_{ijk} + \delta_5 I_{ijk2} I_{ijk3} x_{ijk} + \epsilon_{ijk}$$

$H_0$ : all  $\delta_i$  equal zero ( $i = 1, \dots, 5$ ),  $H_a$ : not all  $\delta_i$  equal zero.

$$SSE(R) = 8.2941, SSE(F) = 6.1765,$$

$$F^* = (2.1176/5) \div (6.1765/24) = 1.646, F(.99; 5, 24) = 3.90.$$

If  $F^* \leq 3.90$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .19

22.16. a. 
$$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \alpha_2 I_{ijk2} + \beta_1 I_{ijk3} + (\alpha\beta)_{11} I_{ijk1} I_{ijk3}$$

$$+ (\alpha\beta)_{21} I_{ijk2} I_{ijk3} + \gamma x_{ijk} + \epsilon_{ijk}$$

$$I_{ijk1} = \begin{cases} 1 & \text{if case from level 1 for factor } A \\ -1 & \text{if case from level 3 for factor } A \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ijk2} = \begin{cases} 1 & \text{if case from level 2 for factor } A \\ -1 & \text{if case from level 3 for factor } A \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ijk3} = \begin{cases} 1 & \text{if case from level 1 for factor } B \\ -1 & \text{if case from level 2 for factor } B \end{cases}$$

$$x_{ijk} = X_{ijk} - \bar{X}_{...} \quad (\bar{X}_{...} = 3.4083)$$

$$\hat{Y} = 23.55556 - 2.15283I_1 + 3.68152I_2 + .20907I_3 - .06009I_1I_3 - .04615I_2I_3 + 1.06122x$$

$$SSE(F) = 8.2941$$

b. Interactions:

$$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \alpha_2 I_{ijk2} + \beta_1 I_{ijk3} + \gamma x_{ijk} + \epsilon_{ijk}$$

$$\hat{Y} = 23.55556 - 2.15400I_1 + 3.67538I_2 + .20692I_3 + 1.07393x$$

$$SSE(R) = 8.4889$$

Factor A:

$$Y_{ijk} = \mu_{..} + \beta_1 I_{ijk3} + (\alpha\beta)_{11} I_{ijk1} I_{ijk3} + (\alpha\beta)_{21} I_{ijk2} I_{ijk3} + \gamma x_{ijk} + \epsilon_{ijk}$$

$$\hat{Y} = 23.55556 + .12982 I_3 + .01136 I_1 I_3 + .06818 I_2 I_3 + 1.52893 x$$

$$SSE(R) = 240.7835$$

Factor B:

$$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \alpha_2 I_{ijk2} + (\alpha\beta)_{11} I_{ijk1} I_{ijk3}$$

$$+ (\alpha\beta)_{21} I_{ijk2} I_{ijk3} + \gamma x_{ijk} + \epsilon_{ijk}$$

$$\hat{Y} = 23.55556 - 2.15487 I_1 + 3.67076 I_2 - .05669 I_1 I_3 - .04071 I_2 I_3 + 1.08348 x$$

$$SSE(R) = 9.8393$$

- c.  $H_0: (\alpha\beta)_{11} = (\alpha\beta)_{21} = 0$ ,  $H_a$ : not both  $(\alpha\beta)_{11}$  and  $(\alpha\beta)_{21}$  equal zero.

$$F^* = (.1948/2) \div (8.2941/29) = .341, F(.95; 2, 29) = 3.33.$$

If  $F^* \leq 3.33$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .714

- d.  $H_0: \alpha_1 = \alpha_2 = 0$ ,  $H_a$ : not both  $\alpha_1$  and  $\alpha_2$  equal zero.

$$F^* = (232.4894/2) \div (8.2941/29) = 406.445, F(.95; 2, 29) = 3.33.$$

If  $F^* \leq 3.33$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

- e.  $H_0: \beta_1 = 0$ ,  $H_a: \beta_1 \neq 0$ .

$$F^* = (1.5452/1) \div (8.2941/29) = 5.403, F(.95; 1, 29) = 4.18.$$

If  $F^* \leq 4.18$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = .027

- f.  $\hat{D}_1 = \hat{\alpha}_1 - \hat{\alpha}_2 = -5.83435$ ,  $\hat{D}_2 = \hat{\alpha}_1 - \hat{\alpha}_3 = 2\hat{\alpha}_1 + \hat{\alpha}_2 = -.62414$ ,

$$\hat{D}_3 = \hat{\alpha}_2 - \hat{\alpha}_3 = 2\hat{\alpha}_2 + \hat{\alpha}_1 = 5.21021, \hat{D}_4 = \hat{\beta}_1 - \hat{\beta}_2 = 2\hat{\beta}_1 = .41814,$$

$$s^2\{\hat{\alpha}_1\} = .01593, s^2\{\hat{\alpha}_2\} = .01708, s\{\hat{\alpha}_1, \hat{\alpha}_2\} = -.00772, s^2\{\hat{\beta}_1\} = .00809,$$

$$s\{\hat{D}_1\} = .22011, s\{\hat{D}_2\} = .22343, s\{\hat{D}_3\} = .23102, s\{\hat{D}_4\} = .17989,$$

$$B = t(.9875; 29) = 2.364$$

$$-5.83435 \pm 2.364(.22011) \quad -6.355 \leq D_1 \leq -5.314$$

$$-.62414 \pm 2.364(.22343) \quad -1.152 \leq D_2 \leq -.096$$

$$5.21021 \pm 2.364(.23102) \quad 4.664 \leq D_3 \leq 5.756$$

$$.41814 \pm 2.364(.17989) \quad -.007 \leq D_4 \leq .843$$

- 22.19. b.  $Y_{ij} = \mu_{..} + \rho_1 I_{ij1} + \rho_2 I_{ij2} + \rho_3 I_{ij3} + \rho_4 I_{ij4} + \rho_5 I_{ij5} + \rho_6 I_{ij6}$

$$+ \rho_7 I_{ij7} + \rho_8 I_{ij8} + \rho_9 I_{ij9} + \tau_1 I_{ij10} + \tau_2 I_{ij11} + \gamma x_{ij} + \epsilon_{ij}$$

$$I_{ij1} = \begin{cases} 1 & \text{if experimental unit from block 1} \\ -1 & \text{if experimental unit from block 10} \\ 0 & \text{otherwise} \end{cases}$$

$I_{ij2}, \dots, I_{ij9}$  are defined similarly

$$I_{ij10} = \begin{cases} 1 & \text{if experimental unit received treatment 1} \\ -1 & \text{if experimental unit received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ij11} = \begin{cases} 1 & \text{if experimental unit received treatment 2} \\ -1 & \text{if experimental unit received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ij} = X_{ij} - \bar{X}_{..} \quad (\bar{X}_{..} = 80.033333)$$

c.  $\hat{Y} = 77.10000 + 4.87199I_1 + 3.87266I_2 + 2.21201I_3 + 3.22003I_4$   
 $+ 1.23474I_5 + .90876I_6 - 1.09124I_7 - 3.74253I_8 - 4.08322I_9$   
 $- 6.50033I_{10} - 2.49993I_{11} + .00201x$

$$SSE(F) = 112.3327$$

d.  $Y_{ij} = \mu_{..} + \rho_1 I_{ij1} + \rho_2 I_{ij2} + \rho_3 I_{ij3} + \rho_4 I_{ij4} + \rho_5 I_{ij5} + \rho_6 I_{ij6}$   
 $+ \rho_7 I_{ij7} + \rho_8 I_{ij8} + \rho_9 I_{ij9} + \gamma x_{ij} + \epsilon_{ij}$

$$\hat{Y} = 77.10000 + 6.71567I_1 + 5.67233I_2 + 3.61567I_3 + 4.09567I_4$$
  
 $+ 1.14233I_5 + .33233I_6 - 1.66767I_7 - 5.33100I_8 - 5.18767I_9 - .13000x$

$$SSE(R) = 1,404.5167$$

e.  $H_0: \tau_1 = \tau_2 = 0, H_a: \text{not both } \tau_1 \text{ and } \tau_2 \text{ equal zero.}$

$$F^* = (1,292.18/2) \div (112.3327/17) = 97.777, F(.95; 2, 17) = 3.59.$$

If  $F^* \leq 3.59$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P\text{-value} = 0+$

f.  $\hat{\tau}_1 = -6.50033, \hat{\tau}_2 = -2.49993, \hat{L} = -4.0004, L^2\{\hat{\tau}_1\} = .44162, s^2\{\hat{\tau}_2\} = .44056,$   
 $s\{\hat{\tau}_1, \hat{\tau}_2\} = -.22048, s\{\hat{L}\} = 1.1503, t(.975; 17) = 2.11,$   
 $-4.0004 \pm 2.11(1.1503), -6.43 \leq L \leq -1.57$

22.21. a.

Source	<i>SS</i>	<i>df</i>	<i>MS</i>
Between treatments	25.5824	2	12.7912
Error	1.4650	24	.0610
Total	27.0474	26	

b. Covariance:  $MSE = .0573, \hat{\gamma} = 1.11417$

# Chapter 23

## TWO-FACTOR STUDIES – UNEQUAL SAMPLE SIZES

23.4. a.  $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \beta_1 X_{ijk3} + \beta_2 X_{ijk4}$

$$+ (\alpha\beta)_{11} X_{ijk1} X_{ijk3} + (\alpha\beta)_{12} X_{ijk1} X_{ijk4}$$

$$+ (\alpha\beta)_{21} X_{ijk2} X_{ijk3} + (\alpha\beta)_{22} X_{ijk2} X_{ijk4} + \epsilon_{ijk}$$

$$X_{ijk1} = \begin{array}{l} 1 \text{ if case from level 1 for factor } A \\ -1 \text{ if case from level 3 factor } A \\ 0 \text{ otherwise} \end{array}$$

$$X_{ijk2} = \begin{array}{l} 1 \text{ if case from level 2 for factor } A \\ -1 \text{ if case from level 3 for factor } A \\ 0 \text{ otherwise} \end{array}$$

$$X_{ijk3} = \begin{array}{l} 1 \text{ if case from level 1 for factor } B \\ -1 \text{ if case from level 3 for factor } B \\ 0 \text{ otherwise} \end{array}$$

$$X_{ijk4} = \begin{array}{l} 1 \text{ if case from level 2 for factor } B \\ -1 \text{ if case from level 3 for factor } B \\ 0 \text{ otherwise} \end{array}$$

b. **Y** entries: in order  $Y_{111}, \dots, Y_{114}, Y_{121}, \dots, Y_{124}, Y_{131}, \dots, Y_{134}, Y_{211}, \dots$

**$\beta$**  entries:  $\mu_{..}, \alpha_1, \alpha_2, \beta_1, \beta_2, (\alpha\beta)_{11}, (\alpha\beta)_{12}, (\alpha\beta)_{21}, (\alpha\beta)_{22}$

**X** entries:

<i>A</i>	<i>B</i>	Freq.	$X_1$	$X_2$	$X_3$	$X_4$	$X_1X_3$	$X_1X_4$	$X_2X_3$	$X_2X_4$	
1	1	4	1	1	0	1	0	1	0	0	0
1	2	4	1	1	0	0	1	0	1	0	0
1	3	4	1	1	0	-1	-1	-1	-1	0	0
2	1	4	1	0	1	1	0	0	0	1	0
2	2	4	1	0	1	0	1	0	0	0	1
2	3	4	1	0	1	-1	-1	0	0	-1	-1
3	1	4	1	-1	-1	1	0	-1	0	-1	0
3	2	4	1	-1	-1	0	1	0	-1	0	-1
3	3	4	1	-1	-1	-1	-1	1	1	1	1

c.  $\mathbf{X}\boldsymbol{\beta}$  entries:

<i>A</i>	<i>B</i>	
1	1	$\mu_{..} + \alpha_1 + \beta_1 + (\alpha\beta)_{11}$
1	2	$\mu_{..} + \alpha_1 + \beta_2 + (\alpha\beta)_{12}$
1	3	$\mu_{..} + \alpha_1 - \beta_1 - \beta_2 - (\alpha\beta)_{11} - (\alpha\beta)_{12} = \mu_{..} + \alpha_1 + \beta_3 + (\alpha\beta)_{13}$
2	1	$\mu_{..} + \alpha_2 + \beta_1 + (\alpha\beta)_{21}$
2	2	$\mu_{..} + \alpha_2 + \beta_2 + (\alpha\beta)_{22}$
2	3	$\mu_{..} + \alpha_2 - \beta_1 - \beta_2 - (\alpha\beta)_{21} - (\alpha\beta)_{22} = \mu_{..} + \alpha_2 + \beta_3 + (\alpha\beta)_{23}$
3	1	$\mu_{..} - \alpha_1 - \alpha_2 + \beta_1 - (\alpha\beta)_{11} - (\alpha\beta)_{21} = \mu_{..} + \alpha_3 + \beta_1 + (\alpha\beta)_{31}$
3	2	$\mu_{..} - \alpha_1 - \alpha_2 + \beta_2 - (\alpha\beta)_{12} - (\alpha\beta)_{22} = \mu_{..} + \alpha_3 + \beta_2 + (\alpha\beta)_{32}$
3	3	$\mu_{..} - \alpha_1 - \alpha_2 - \beta_1 - \beta_2 + (\alpha\beta)_{11} + (\alpha\beta)_{12} + (\alpha\beta)_{21} + (\alpha\beta)_{22}$ $= \mu_{..} + \alpha_3 + \beta_3 + (\alpha\beta)_{33}$

d.  $\hat{Y} = 7.18333 - 3.30000X_1 + .65000X_2 - 2.55000X_3 + .75000X_4$   
 $+ 1.14167X_1X_3 - .03333X_1X_4 + .16667X_2X_3 + .34167X_2X_4$

$\alpha_1 = \mu_{1.} - \mu_{..}$

e.

Source	<i>SS</i>	<i>df</i>
Regression	373.125	8
$X_1$	212.415	1] <i>A</i>
$X_2 \mid X_1$	7.605	1] <i>A</i>
$X_3 \mid X_1, X_2$	113.535	1] <i>B</i>
$X_4 \mid X_1, X_2, X_3$	10.125	1] <i>B</i>
$X_1X_3 \mid X_1, X_2, X_3, X_4$	26.7806	1] <i>AB</i>
$X_1X_4 \mid X_1, X_2, X_3, X_4, X_1X_3$	.2269	1] <i>AB</i>
$X_2X_3 \mid X_1, X_2, X_3, X_4, X_1X_3, X_1X_4$	1.3669	1] <i>AB</i>
$X_2X_4 \mid X_1, X_2, X_3, X_4, X_1X_3, X_1X_4, X_2X_3$	1.0506	1] <i>AB</i>
Error	1.625	27
Total	374.730	35

Yes.

f. See Problem 19.15c and d.

23.6. a.  $Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$

$Y_{ijk} = \mu_{..} + \alpha_1X_{ijk1} + \alpha_2X_{ijk2} + \beta_1X_{ijk3} + (\alpha\beta)_{11}X_{ijk1}X_{ijk3}$

$$+(\alpha\beta)_{21}X_{ijk2}X_{ijk3} + \epsilon_{ijk}$$

$$X_{ijk1} = \begin{cases} 1 & \text{if case from level 1 for factor } A \\ -1 & \text{if case from level 3 for factor } A \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk2} = \begin{cases} 1 & \text{if case from level 2 for factor } A \\ -1 & \text{if case from level 3 for factor } A \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk3} = \begin{cases} 1 & \text{if case from level 1 for factor } B \\ -1 & \text{if case from level 2 for factor } B \end{cases}$$

b.  $\beta$  entries:  $\mu_{..}$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $\beta_1$ ,  $(\alpha\beta)_{11}$ ,  $(\alpha\beta)_{21}$

$\mathbf{X}$  entries:

$A$	$B$	Freq.	$X_1$	$X_2$	$X_3$	$X_1X_3$	$X_2X_3$
1	1	6	1	1	0	1	0
1	2	6	1	1	0	-1	0
2	1	5	1	0	1	1	1
2	2	6	1	0	1	-1	-1
3	1	6	1	-1	-1	1	-1
3	2	5	1	-1	-1	-1	1

c.  $\mathbf{X}\beta$  entries:

$A$	$B$	
1	1	$\mu_{..} + \alpha_1 + \beta_1 + (\alpha\beta)_{11}$
1	2	$\mu_{..} + \alpha_1 - \beta_1 - (\alpha\beta)_{11} = \mu_{..} + \alpha_1 + \beta_2 + (\alpha\beta)_{12}$
2	1	$\mu_{..} + \alpha_2 + \beta_1 + (\alpha\beta)_{21}$
2	2	$\mu_{..} + \alpha_2 - \beta_1 - (\alpha\beta)_{21} = \mu_{..} + \alpha_2 + \beta_2 + (\alpha\beta)_{22}$
3	1	$\mu_{..} - \alpha_1 - \alpha_2 + \beta_1 - (\alpha\beta)_{11} - (\alpha\beta)_{21} = \mu_{..} + \alpha_3 + \beta_1 + (\alpha\beta)_{31}$
3	2	$\mu_{..} - \alpha_1 - \alpha_2 - \beta_1 + (\alpha\beta)_{11} + (\alpha\beta)_{21} = \mu_{..} + \alpha_3 + \beta_2 + (\alpha\beta)_{32}$

d.  $Y_{ijk} = \mu_{..} + \alpha_1X_{ijk1} + \alpha_2X_{ijk2} + \beta_1X_{ijk3} + \epsilon_{ijk}$

e. Full model:

$$\hat{Y} = 23.56667 - 2.06667X_1 + 4.16667X_2 + .36667X_3 - .20000X_1X_3 - .30000X_2X_3,$$

$$SSE(F) = 71.3333$$

Reduced model:

$$\hat{Y} = 23.59091 - 2.09091X_1 + 4.16911X_2 + .36022X_3,$$

$$SSE(R) = 75.5210$$

$H_0: (\alpha\beta)_{11} = (\alpha\beta)_{21} = 0$ ,  $H_a$ : not both  $(\alpha\beta)_{11}$  and  $(\alpha\beta)_{21}$  equal zero.

$$F^* = (4.1877/2) \div (71.3333/28) = .82, F(.95; 2, 28) = 3.34.$$

If  $F^* \leq 3.34$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .45

f. A effects:

$$\hat{Y} = 23.50000 + .17677X_3 - .01010X_1X_3 - .49495X_2X_3,$$

$$SSE(R) = 359.9394$$

$H_0: \alpha_1 = \alpha_2 = 0, H_a: \text{not both } \alpha_1 \text{ and } \alpha_2 \text{ equal zero.}$

$$F^* = (288.6061/2) \div (71.3333/28) = 56.64, F(.95; 2, 28) = 3.34.$$

If  $F^* \leq 3.34$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P\text{-value} = 0+$

B effects:

$$\hat{Y} = 23.56667 - 2.06667X_1 + 4.13229X_2 - .17708X_1X_3 - .31146X_2X_3,$$

$$SSE(R) = 75.8708$$

$H_0: \beta_1 = 0, H_a: \beta_1 \neq 0.$

$$F^* = (4.5375/1) \div (71.3333/28) = 1.78, F(.95; 1, 28) = 4.20.$$

If  $F^* \leq 4.20$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P\text{-value} = .19$

g.  $\hat{D}_1 = \hat{\alpha}_1 - \hat{\alpha}_2 = -6.23334, \hat{D}_2 = \hat{\alpha}_1 - \hat{\alpha}_3 = 2\hat{\alpha}_1 + \hat{\alpha}_2 = .03333, \hat{D}_3 = \hat{\alpha}_2 - \hat{\alpha}_3 = 2\hat{\alpha}_2 + \hat{\alpha}_1 = 6.26667, s^2\{\hat{\alpha}_1\} = .14625, s^2\{\hat{\alpha}_2\} = .15333, s\{\hat{\alpha}_1, \hat{\alpha}_2\} = -.07313, s\{\hat{D}_1\} = .6677, s\{\hat{D}_2\} = .6677, s\{\hat{D}_3\} = .6834, q(.90; 3, 28) = 3.026, T = 2.140$

$$\begin{array}{ll} -6.23334 \pm 2.140(.6677) & -7.662 \leq D_1 \leq -4.804 \\ .03333 \pm 2.140(.6677) & -1.396 \leq D_2 \leq 1.462 \\ 6.26667 \pm 2.140(.6834) & 4.804 \leq D_3 \leq 7.729 \end{array}$$

h.  $\hat{L} = .3\bar{Y}_{12} + .6\bar{Y}_{22} + .1\bar{Y}_{32} = .3(21.33333) + .6(27.66667) + .1(20.60000) = 25.06000, s\{\hat{L}\} = .4429, t(.975; 28) = 2.048, 25.06000 \pm 2.048(.4429), 24.153 \leq L \leq 25.967$

23.7. a.  $Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$

$$\begin{aligned} Y_{ijk} = & \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \beta_1 X_{ijk3} + \beta_2 X_{ijk4} \\ & + (\alpha\beta)_{11} X_{ijk1} X_{ijk3} + (\alpha\beta)_{12} X_{ijk1} X_{ijk4} + (\alpha\beta)_{21} X_{ijk2} X_{ijk3} \\ & + (\alpha\beta)_{22} X_{ijk2} X_{ijk4} + \epsilon_{ijk} \end{aligned}$$

$$X_{ijk1} = \begin{array}{l} 1 \text{ if case from level 1 for factor } A \\ -1 \text{ if case from level 3 for factor } A \\ 0 \text{ otherwise} \end{array}$$

$$X_{ijk2} = \begin{array}{l} 1 \text{ if case from level 2 for factor } A \\ -1 \text{ if case from level 3 for factor } A \\ 0 \text{ otherwise} \end{array}$$

$$X_{ijk3} = \begin{array}{l} 1 \text{ if case from level 1 for factor } B \\ -1 \text{ if case from level 3 for factor } B \\ 0 \text{ otherwise} \end{array}$$

$$X_{ijk4} = \begin{array}{l} 1 \text{ if case from level 2 for factor } B \\ -1 \text{ if case from level 3 for factor } B \\ 0 \text{ otherwise} \end{array}$$

b.  $\beta$  entries:  $\mu_{..}, \alpha_1, \alpha_2, \beta_1, \beta_2, (\alpha\beta)_{11}, (\alpha\beta)_{12}, (\alpha\beta)_{21}, (\alpha\beta)_{22}$

$\mathbf{X}$  entries:

A	B	Freq.	$X_1$	$X_2$	$X_3$	$X_4$	$X_1X_3$	$X_1X_4$	$X_2X_3$	$X_2X_4$
1	1	3	1	1	0	1	0	1	0	0
1	2	4	1	1	0	0	1	0	1	0
1	3	4	1	1	0	-1	-1	-1	-1	0
2	1	4	1	0	1	1	0	0	0	1
2	2	2	1	0	1	0	1	0	0	1
2	3	4	1	0	1	-1	-1	0	0	-1
3	1	4	1	-1	-1	1	0	-1	0	-1
3	2	4	1	-1	-1	0	1	0	-1	0
3	3	4	1	-1	-1	1	-1	1	1	1

c.  $\mathbf{X}\boldsymbol{\beta}$  entries:

A	B	
1	1	$\mu_{..} + \alpha_1 + \beta_1 + (\alpha\beta)_{11}$
1	2	$\mu_{..} + \alpha_1 + \beta_2 + (\alpha\beta)_{12}$
1	3	$\mu_{..} + \alpha_1 - \beta_1 - \beta_2 - (\alpha\beta)_{11} - (\alpha\beta)_{12} = \mu_{..} + \alpha_1 + \beta_3 + (\alpha\beta)_{13}$
2	1	$\mu_{..} + \alpha_2 + \beta_1 + (\alpha\beta)_{21}$
2	2	$\mu_{..} + \alpha_2 + \beta_2 + (\alpha\beta)_{22}$
2	3	$\mu_{..} + \alpha_2 - \beta_1 - \beta_2 - (\alpha\beta)_{21} - (\alpha\beta)_{22} = \mu_{..} + \alpha_2 + \beta_3 + (\alpha\beta)_{23}$
3	1	$\mu_{..} - \alpha_1 - \alpha_2 + \beta_1 - (\alpha\beta)_{11} - (\alpha\beta)_{21} = \mu_{..} + \alpha_3 + \beta_1 + (\alpha\beta)_{31}$
3	2	$\mu_{..} - \alpha_1 - \alpha_2 + \beta_2 - (\alpha\beta)_{12} - (\alpha\beta)_{22} = \mu_{..} + \alpha_3 + \beta_2 + (\alpha\beta)_{32}$
3	3	$\mu_{..} - \alpha_1 - \alpha_2 - \beta_1 - \beta_2 + (\alpha\beta)_{11} + (\alpha\beta)_{12} + (\alpha\beta)_{21} + (\alpha\beta)_{22}$ $= \mu_{..} + \alpha_3 + \beta_3 + (\alpha\beta)_{33}$

d.  $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \beta_1 X_{ijk3} + \beta_2 X_{ijk4} + \epsilon_{ijk}$

e. Full model:

$$\hat{Y} = 7.18704 - 3.28426X_1 + .63796X_2 - 2.53426X_3 + .73796X_4 \\ + 1.16481X_1X_3 - .04074X_1X_4 + .15926X_2X_3 + .33704X_2X_4,$$

$$SSE(F) = 1.5767$$

Reduced model:

$$\hat{Y} = 7.12711 - 3.33483X_1 + .62861X_2 - 2.58483X_3 + .72861X_4,$$

$$SSE(R) = 29.6474$$

$H_0$ : all  $(\alpha\beta)_{ij}$  equal zero,  $H_a$ : not all  $(\alpha\beta)_{ij}$  equal zero.

$$F^* = (28.0707/4) \div (1.5767/24) = 106.82, F(.95; 4, 24) = 2.78.$$

If  $F^* \leq 2.78$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

f.  $\bar{Y}_{11.} = 2.5333, \bar{Y}_{12.} = 4.6000, \bar{Y}_{13.} = 4.57500, \bar{Y}_{21.} = 5.45000, \bar{Y}_{22.} = 8.90000,$   
 $\bar{Y}_{23.} = 9.12500, \bar{Y}_{31.} = 5.97500, \bar{Y}_{32.} = 10.27500, \bar{Y}_{33.} = 13.25000, \hat{L}_1 = 2.0542,$   
 $\hat{L}_2 = 3.5625, \hat{L}_3 = 5.7875, \hat{L}_4 = 1.5083, \hat{L}_5 = 3.7333, \hat{L}_6 = 2.2250, s\{\hat{L}_1\} = .1613,$   
 $s\{\hat{L}_2\} = .1695, s\{\hat{L}_3\} = .1570, s\{\hat{L}_4\} = .2340, s\{\hat{L}_5\} = .2251, s\{\hat{L}_6\} = .2310,$   
 $F(.90; 8, 24) = 1.94, S = 3.9395$



$$\begin{array}{ll}
2.0542 \pm 3.9395(.1613) & 1.419 \leq L_1 \leq 2.690 \\
3.5625 \pm 3.9395(.1695) & 2.895 \leq L_2 \leq 4.230 \\
5.7875 \pm 3.9395(.1570) & 5.169 \leq L_3 \leq 6.406 \\
1.5083 \pm 3.9395(.2340) & .586 \leq L_4 \leq 2.430 \\
3.7333 \pm 3.9395(.2251) & 2.846 \leq L_5 \leq 4.620 \\
2.2250 \pm 3.9395(.2310) & 1.315 \leq L_6 \leq 3.135
\end{array}$$

23.12. a. See Problem 19.14a.  $\hat{D}_1 = \bar{Y}_{13} - \bar{Y}_{11} = 2.100$ ,  $\hat{D}_2 = \bar{Y}_{23} - \bar{Y}_{21} = 3.675$ ,

$$\begin{aligned}
\hat{D}_3 &= \bar{Y}_{33} - \bar{Y}_{31} = 7.275, \hat{L}_1 = \hat{D}_1 - \hat{D}_2 = -1.575, \hat{L}_2 = \hat{D}_1 - \hat{D}_3 = -5.175, \\
MSE &= .06406, s\{\hat{D}_i\} = .1790 \quad (i = 1, 2, 3), s\{\hat{L}_i\} = .2531 \quad (i = 1, 2), B = \\
t(.99; 24) &= 2.492
\end{aligned}$$

$$\begin{array}{ll}
2.100 \pm 2.492(.1790) & 1.654 \leq D_1 \leq 2.546 \\
3.675 \pm 2.492(.1790) & 3.229 \leq D_2 \leq 4.121 \\
7.275 \pm 2.492(.1790) & 6.829 \leq D_3 \leq 7.721 \\
-1.575 \pm 2.492(.2531) & -2.206 \leq L_1 \leq -.944 \\
-5.175 \pm 2.492(.2531) & -5.806 \leq L_2 \leq -4.544
\end{array}$$

b.  $H_0: \mu_{12} - \mu_{13} = 0$ ,  $H_a: \mu_{12} - \mu_{13} \neq 0$ .  $\hat{D} = \bar{Y}_{12} - \bar{Y}_{13} = .025$ ,  $s\{\hat{D}\} = .1790$ ,  $t^* = .025/.1790 = .14$ ,  $t(.99; 24) = 2.492$ . If  $|t^*| \leq 2.492$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

$H_0: \mu_{32} - \mu_{33} = 0$ ,  $H_a: \mu_{32} - \mu_{33} \neq 0$ .  $\hat{D} = \bar{Y}_{32} - \bar{Y}_{33} = -2.975$ ,  $s\{\hat{D}\} = .1790$ ,  $t^* = -2.975/.1790 = -16.62$ ,  $t(.99; 24) = 2.492$ . If  $|t^*| \leq 2.492$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $\alpha \leq .04$

23.14. a. See Problem 19.20a.  $\hat{D}_1 = \bar{Y}_{12} - \bar{Y}_{13} = 46.0$ ,  $\hat{D}_2 = \bar{Y}_{22} - \bar{Y}_{23} = 6.0$ ,

$$\begin{aligned}
\hat{L}_1 &= \hat{D}_1 - \hat{D}_2 = 40.0, MSE = 88.50, s\{\hat{D}_1\} = s\{\hat{D}_2\} = 6.652, s\{\hat{L}_1\} = 9.407, \\
B &= t(.99167; 15) = 2.694
\end{aligned}$$

$$\begin{array}{ll}
46.0 \pm 2.694(6.652) & 28.080 \leq D_1 \leq 63.920 \\
6.0 \pm 2.694(6.652) & -11.920 \leq D_2 \leq 23.920 \\
40.0 \pm 2.694(9.407) & 14.658 \leq L_1 \leq 65.342
\end{array}$$

b.  $H_0: \mu_{22} - \mu_{23} \leq 0$ ,  $H_a: \mu_{22} - \mu_{23} > 0$ .  $\hat{D} = \bar{Y}_{22} - \bar{Y}_{23} = 6.0$ ,  $s\{\hat{D}\} = 6.652$ ,  $t^* = 6.0/6.652 = .90$ ,  $t(.95; 15) = 1.753$ . If  $t^* \leq 1.753$  conclude  $H_0$  otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .19

23.16. a.  $Y_{ij} = \mu_{..} + \rho_1 X_{ij1} + \rho_2 X_{ij2} + \rho_3 X_{ij3} + \rho_4 X_{ij4} + \rho_5 X_{ij5} + \rho_6 X_{ij6}$

$$+ \rho_7 X_{ij7} + \rho_8 X_{ij8} + \rho_9 X_{ij9} + \tau_1 X_{ij10} + \tau_2 X_{ij11} + \epsilon_{ij}$$

$$X_{ij1} = \begin{cases} 1 & \text{if experimental unit from block 1} \\ -1 & \text{if experimental unit from block 10} \\ 0 & \text{otherwise} \end{cases}$$

$X_{ij2}, \dots, X_{ij9}$  are defined similarly

$$X_{ij10} = \begin{cases} 1 & \text{if experimental unit received treatment 1} \\ -1 & \text{if experimental unit received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ij11} = \begin{cases} 1 & \text{if experimental unit received treatment 2} \\ -1 & \text{if experimental unit received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

b.  $\hat{Y} = 77.10000 + 4.90000X_1 + 3.90000X_2 + 2.23333X_3 + 3.23333X_4 + 1.23333X_5 + .90000X_6 - 1.10000X_7 - 3.76667X_8 - 4.10000X_9 - 6.50000X_{10} - 2.50000X_{11}$

c.

Source	<i>SS</i>	<i>df</i>	<i>MS</i>
Regression	1,728.3667	1	157.1242
$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9$	433.3667	9	48.1519
$X_{10}, X_{11}   X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9$	1,295.0000	2	647.5000
Error	112.3333	18	6.2407
Total	1,840.7000	29	

d.  $H_0: \tau_1 = \tau_2 = 0, H_a: \text{not both } \tau_1 \text{ and } \tau_2 \text{ equal zero.}$

$$F^* = (1,295.0000/2) \div (112.3333/18) = 103.754, F(.95; 2, 18) = 3.55.$$

If  $F^* \leq 3.55$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .

23.18. a.  $Y_{ij} = \mu_{..} + \rho_i + \tau_j + \epsilon_{ij}$

$$Y_{ij} = \mu_{..} + \rho_1 X_{ij1} + \rho_2 X_{ij2} + \rho_3 X_{ij3} + \rho_4 X_{ij4} + \rho_5 X_{ij5} + \rho_6 X_{ij6} + \rho_7 X_{ij7} + \rho_8 X_{ij8} + \rho_9 X_{ij9} + \tau_1 X_{ij10} + \tau_2 X_{ij11} + \epsilon_{ij}$$

$$X_{ij1} = \begin{cases} 1 & \text{if experimental unit from block 1} \\ -1 & \text{if experimental unit from block 10} \\ 0 & \text{otherwise} \end{cases}$$

$X_{ij2}, \dots, X_{ij9}$  are defined similarly

$$X_{ij10} = \begin{cases} 1 & \text{if experimental unit received treatment 1} \\ -1 & \text{if experimental unit received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ij11} = \begin{cases} 1 & \text{if experimental unit received treatment 2} \\ -1 & \text{if experimental unit received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

b.  $Y_{ij} = \mu_{..} + \rho_1 X_{ij1} + \rho_2 X_{ij2} + \rho_3 X_{ij3} + \rho_4 X_{ij4} + \rho_5 X_{ij5} + \rho_6 X_{ij6}$

$$+\rho_7 X_{ij7} + \rho_8 X_{ij8} + \rho_9 X_{ij9} + \epsilon_{ij}$$

c. Full model:  $\hat{Y} = 77.15556 + 4.84444X_1 + 4.40000X_2 + 2.17778X_3$   
 $+3.17778X_4 + 1.17778X_5 + .84444X_6 - 1.15556X_7$   
 $-3.82222X_8 - 4.15556X_9 - 6.55556X_{10} - 2.55556X_{11}$

$$SSE(F) = 110.6667$$

Reduced model:  $\hat{Y} = 76.70000 + 5.30000X_1 + .30000X_2 + 2.63333X_3$   
 $+3.63333X_4 + 1.63333X_5 + 1.30000X_6 - .70000X_7$   
 $-3.36667X_8 - 3.70000X_9$

$$SSE(R) = 1,311.3333$$

$H_0$ :  $\tau_1 = \tau_2 = 0$ ,  $H_a$ : not both  $\tau_1$  and  $\tau_2$  equal zero.

$$F^* = (1,200.6666/2) \div (110.6667/17) = 92.22, F(.95; 2, 17) = 3.59.$$

If  $F^* \leq 3.59$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .

d.  $\hat{L} = \hat{\tau}_2 - \hat{\tau}_3 = 2\hat{\tau}_2 + \hat{\tau}_1 = -11.66667$ ,  $s^2\{\hat{\tau}_i\} = .44604$  ( $i = 1, 2$ ),  $s\{\hat{\tau}_1, \hat{\tau}_2\} =$   
 $-.20494$ ,  $s\{\hat{L}\} = 1.1876$ ,  $t(.975; 17) = 2.11$ ,  
 $-11.66667 \pm 2.11(1.1876)$ ,  $-14.17 \leq L \leq -9.16$

23.20. See Problem 19.10a.  $L_1 = .3\mu_{11} + .6\mu_{21} + .1\mu_{31}$ ,  $L_2 = .3\mu_{12} + .6\mu_{22} + .1\mu_{32}$ .

$H_0$ :  $L_1 = L_2$ ,  $H_a$ :  $L_1 \neq L_2$ .

$$\hat{L}_1 - \hat{L}_2 = 25.43332 - 25.05001 = .38331, MSE = 2.3889, s\{\hat{L}_1 - \hat{L}_2\} = .6052,$$

$$t^* = .38331/.6052 = .63, t(.975; 30) = 2.042.$$

If  $|t^*| \leq 2.042$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .53

23.23.  $H_0$ :  $\frac{4\mu_{11}+4\mu_{12}+2\mu_{13}}{10} = \frac{4\mu_{21}+4\mu_{22}+3\mu_{23}}{11}$ ,  $H_a$ : equality does not hold.

$$\bar{Y}_{...} = 93.714, \bar{Y}_{1..} = 143, \bar{Y}_{2..} = 48.91$$

$$SSA = 10(143 - 93.714)^2 + 11(48.91 - 93.714)^2 = 46,372$$

$$F^* = (46,372/1) \div (1,423.1667/15) = 488.8, F(.99; 1, 15) = 8.68.$$

If  $F^* \leq 8.68$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

# Chapter 24

## MULTIFACTOR STUDIES

24.6. a.  $e_{ijkm}$ :

$k = 1$			$k = 2$		
$i$	$j = 1$	$j = 2$	$i$	$j = 1$	$j = 2$
1	3.7667	1.1667	1	-.5000	-1.0333
	-3.9333	-1.6333		.4000	1.3667
	.1667	.4667		.1000	-.3333
2	-1.7000	-.8333	2	1.1333	-.5667
	1.1000	1.3667		-1.6667	1.7333
	.6000	-.5333		.5333	-1.1667

b.  $r = .973$

24.7. a.  $\bar{Y}_{111.} = 36.1333, \bar{Y}_{112.} = 56.5000, \bar{Y}_{121.} = 52.3333, \bar{Y}_{122.} = 71.9333,$   
 $\bar{Y}_{211.} = 46.9000, \bar{Y}_{212.} = 68.2667, \bar{Y}_{221.} = 64.1333, \bar{Y}_{222.} = 83.4667$

b.

Source	$SS$	$df$	$MS$
Between treatments	4,772.25835	7	681.75119
$A$ (chemical)	788.90667	1	788.90667
$B$ (temperature)	1,539.20167	1	1,539.20167
$C$ (time)	2,440.16667	1	2,440.16667
$AB$ interactions	.24000	1	.24000
$AC$ interactions	.20167	1	.20167
$BC$ interactions	2.94000	1	2.94000
$ABC$ interactions	.60167	1	.60167
Error	53.74000	16	3.35875
Total	4,825.99835	23	

c.  $H_0$ : all  $(\alpha\beta\gamma)_{ijk}$  equal zero,  $H_a$ : not all  $(\alpha\beta\gamma)_{ijk}$  equal zero.  $F^* = .60167/3.35875 = .18, F(.975; 1, 16) = 6.12$ . If  $F^* \leq 6.12$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .68

d.  $H_0$ : all  $(\alpha\beta)_{ij}$  equal zero,  $H_a$ : not all  $(\alpha\beta)_{ij}$  equal zero.  $F^* = .24000/3.35875 = .07, F(.975; 1, 16) = 6.12$ . If  $F^* \leq 6.12$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .79

$H_0$ : all  $(\alpha\gamma)_{ik}$  equal zero,  $H_a$ : not all  $(\alpha\gamma)_{ik}$  equal zero.  $F^* = .20167/3.35875 = .06$ ,  $F(.975; 1, 16) = 6.12$ . If  $F^* \leq 6.12$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .81

$H_0$ : all  $(\beta\gamma)_{jk}$  equal zero,  $H_a$ : not all  $(\beta\gamma)_{jk}$  equal zero.  $F^* = 2.94000/3.35875 = .875$ ,  $F(.975; 1, 16) = 6.12$ . If  $F^* \leq 6.12$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .36

e.  $H_0$ : all  $\alpha_i$  equal zero ( $i = 1, 2$ ),  $H_a$ : not all  $\alpha_i$  equal zero.  $F^* = 788.90667/3.35875 = 234.88$ ,  $F(.975; 1, 16) = 6.12$ . If  $F^* \leq 6.12$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

$H_0$ : all  $\beta_j$  equal zero ( $j = 1, 2$ ),  $H_a$ : not all  $\beta_j$  equal zero.  $F^* = 1,539.20167/3.35875 = 458.27$ ,  $F(.975; 1, 16) = 6.12$ . If  $F^* \leq 6.12$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

$H_0$ : all  $\gamma_k$  equal zero ( $k = 1, 2$ ),  $H_a$ : not all  $\gamma_k$  equal zero.  $F^* = 2,440.1667/3.35875 = 726.51$ ,  $F(.975; 1, 16) = 6.12$ . If  $F^* \leq 6.12$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

f.  $\alpha \leq .1624$

24.8. a.  $\hat{D}_1 = 65.69167 - 54.22500 = 11.46667$ ,  $\hat{D}_2 = 67.96667 - 51.95000 = 16.01667$ ,  
 $\hat{D}_3 = 70.04167 - 49.87500 = 20.16667$ ,  $MSE = 3.35875$ ,  
 $s\{\hat{D}_i\} = .7482$  ( $i = 1, 2, 3$ ),  $B = t(.99167; 16) = 2.673$

$$\begin{aligned} 11.46667 \pm 2.673(.7482) & \quad 9.467 \leq D_1 \leq 13.467 \\ 16.01667 \pm 2.673(.7482) & \quad 14.017 \leq D_2 \leq 18.017 \\ 20.16667 \pm 2.673(.7482) & \quad 18.167 \leq D_3 \leq 22.167 \end{aligned}$$

b.  $\bar{Y}_{222} = 83.46667$ ,  $s\{\bar{Y}_{222}\} = 1.0581$ ,  $t(.975; 16) = 2.120$ ,  
 $83.46667 \pm 2.120(1.0581)$ ,  $81.2235 \leq \mu_{222} \leq 85.7098$

24.15. a.  $Y_{ijkm} = \mu_{...} + \alpha_1 X_{ijkm1} + \beta_1 X_{ijkm2} + \gamma_1 X_{ijkm3} + (\alpha\beta)_{11} X_{ijkm1} X_{ijkm2} + (\alpha\gamma)_{11} X_{ijkm1} X_{ijkm3}$   
 $+ (\beta\gamma)_{11} X_{ijkm2} X_{ijkm3} + (\alpha\beta\gamma)_{111} X_{ijkm1} X_{ijkm2} X_{ijkm3} + \epsilon_{ijkm}$

$$X_{ijk1} = \begin{aligned} & 1 \text{ if case from level 1 for factor } A \\ & -1 \text{ if case from level 2 for factor } A \end{aligned}$$

$$X_{ijk2} = \begin{aligned} & 1 \text{ if case from level 1 for factor } B \\ & -1 \text{ if case from level 2 for factor } B \end{aligned}$$

$$X_{ijk3} = \begin{aligned} & 1 \text{ if case from level 1 for factor } C \\ & -1 \text{ if case from level 2 for factor } C \end{aligned}$$

b.  $Y_{ijkm} = \mu_{...} + \beta_1 X_{ijkm2} + \gamma_1 X_{ijkm3} + (\alpha\beta)_{11} X_{ijkm1} X_{ijkm2} + (\alpha\gamma)_{11} X_{ijkm1} X_{ijkm3}$   
 $+ (\beta\gamma)_{11} X_{ijkm2} X_{ijkm3} + (\alpha\beta\gamma)_{111} X_{ijkm1} X_{ijkm2} X_{ijkm3} + \epsilon_{ijkm}$

c. Full model:

$$\begin{aligned} \hat{Y} = & 60.01667 - 5.67500X_1 - 8.06667X_2 - 10.02500X_3 + .04167X_1X_2 \\ & + .15000X_1X_3 - .40833X_2X_3 + .10000X_1X_2X_3, \end{aligned}$$

$$SSE(F) = 49.4933$$

Reduced model:

$$\hat{Y} = 61.15167 - 9.20167X_2 - 8.89000X_3 - 1.09333X_1X_2 + 1.28500X_1X_3 \\ - 1.54333X_2X_3 - 1.03500X_1X_2X_3,$$

$$SSE(R) = 667.8413$$

$$H_0: \alpha_1 = 0, H_a: \alpha_1 \neq 0.$$

$$F^* = (618.348/1) \div (49.4933/14) = 174.91, F(.975; 1, 14) = 6.298.$$

If  $F^* \leq 6.298$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

- d.  $\hat{D} = \hat{\mu}_{2..} - \hat{\mu}_{1..} = \hat{\alpha}_2 - \hat{\alpha}_1 = -2\hat{\alpha}_1 = 11.35000, s^2\{\hat{\alpha}_1\} = .18413, s\{\hat{D}\} = .8582,$   
 $t(.975; 14) = 2.145,$   
 $11.35000 \pm 2.145(.8582), 9.509 \leq D \leq 13.191$

24.17.  $\frac{2\sqrt{n}}{1.8} = 4.1475, n = 14$



# Chapter 25

## RANDOM AND MIXED EFFECTS MODELS

25.5. b.  $H_0: \sigma_\mu^2 = 0, H_a: \sigma_\mu^2 > 0. F^* = .45787/.03097 = 14.78, F(.95; 5, 114) = 2.29.$

If  $F^* \leq 2.29$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

c.  $\bar{Y}_.. = .22767, n_T = 120, s\{\bar{Y}_.. \} = .06177, t(.975; 5) = 2.571,$   
 $.22767 \pm 2.571(.06177), .0689 \leq \mu. \leq .3865$

25.6. a.  $F(.025; 5, 114) = .1646, F(.975; 5, 114) = 2.680, L = .22583, U = 4.44098$

$$.1842 \leq \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma^2} \leq .8162$$

b.  $\chi^2(.025; 114) = 90.351, \chi^2(.975; 114) = 145.441, .02427 \leq \sigma^2 \leq .03908$

c.  $s_\mu^2 = .02135$

d. Satterthwaite:

$$df = (ns_\mu^2)^2 \div [(MSTR)^2/(r-1) + (MSE)^2/r(n-1)]$$
$$= [20(.02135)]^2 \div [(.45787)^2/5 + (.03907)^2/6(19)] = 4.35,$$

$$\chi^2(.025; 4) = .484, \chi^2(.975; 4) = 11.143$$

$$.0083 = \frac{4.35(.02135)}{11.143} \leq \sigma_\mu^2 \leq \frac{4.35(.02135)}{.484} = .192$$

$MLS: c_1 = .05, c_2 = -.05, MS_1 = .45787, MS_2 = .03907, df_1 = 5, df_2 = 114,$   
 $F_1 = F(.975; 5, \infty) = 2.57, F_2 = F(.975; 114, \infty) = 1.28, F_3 = F(.975; \infty, 5) =$   
 $6.02, F_4 = F(.975; \infty, 114) = 1.32, F_5 = F(.975; 5, 114) = 2.68, F_6 = F(.975; 114, 5) =$   
 $6.07, G_1 = .6109, G_2 = .2188, G_3 = .0147, G_4 = -.2076, H_L = .014, H_U = .115,$   
 $.02135 - .014, .02135 + .115, .0074 \leq \sigma_\mu^2 \leq .1364$

25.16. a.  $H_0: \sigma_{\alpha\beta}^2 = 0, H_a: \sigma_{\alpha\beta}^2 > 0. F^* = 303.822/52.011 = 5.84, F(.99; 4, 36) = 3.89.$

If  $F^* \leq 3.89$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = .001

b.  $s_{\alpha\beta}^2 = 50.362$

c.  $H_0: \sigma_\alpha^2 = 0, H_a: \sigma_\alpha^2 > 0. F^* = 12.289/52.011 = .24, F(.99; 2, 36) = 5.25.$

If  $F^* \leq 5.25$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .



d.  $H_0$ : all  $\beta_j$  equal zero ( $j = 1, 2, 3$ ),  $H_a$ : not all  $\beta_j$  equal zero.

$$F^* = 14.156/303.822 = .047, F(.99; 2, 4) = 18.0.$$

If  $F^* \leq 18.0$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

e.  $\bar{Y}_{.1} = 56.133, \bar{Y}_{.2} = 56.600, \bar{Y}_{.3} = 54.733, \hat{D}_1 = \bar{Y}_{.1} - \bar{Y}_{.2} = -.467, \hat{D}_2 = \bar{Y}_{.1} - \bar{Y}_{.3} = -1.400, \hat{D}_3 = \bar{Y}_{.2} - \bar{Y}_{.3} = 1.867, s\{\hat{D}_i\} = 6.3647 (i = 1, 2, 3), q(.95; 3, 4) = 5.04, T = 3.5638$

$$\begin{aligned} &-.467 \pm 3.5638(6.3647) & -23.150 \leq D_1 \leq 22.216 \\ &-1.400 \pm 3.5638(6.3647) & -24.083 \leq D_2 \leq 21.283 \\ &1.867 \pm 3.5638(6.3647) & -20.816 \leq D_3 \leq 24.550 \end{aligned}$$

f.  $\hat{\mu}_{.1} = 56.1333, MSA = 12.28889, MSAB = 303.82222, s^2\{\hat{\mu}_{.1}\} = (2/45)(303.82222) + (1/45)(12.28889) = 13.7763, s\{\hat{\mu}_{.1}\} = 3.712, df = (13.7763)^2 \div \{[(2/45)(303.82222)]^2/4 + [(1/45)(12.28889)]^2/2\} = 4.16, t(.995; 4) = 4.60,$

$$56.1333 \pm 4.60(3.712), 39.06 \leq \mu_{.1} \leq 73.21$$

g.  $MSA = 12.28889, MSE = 52.01111, s_\alpha^2 = (MSA - MSE)/nb = -2.648, c_1 = 1/15, c_2 = -1/15, df_1 = 2, df_2 = 36, F_1 = F(.995; 2, \infty) = 5.30, F_2 = F(.995; 36, \infty) = 1.71, F_3 = F(.995; \infty, 2) = 200, F_4 = F(.995; \infty, 36) = 2.01, F_5 = F(.995; 2, 36) = 6.16, F_6 = F(.995; 36, 2) = 199.5, G_1 = .8113, G_2 = .4152, G_3 = .1022, G_4 = -35.3895, H_L = 3.605, H_U = 162.730, -2.648 - 3.605, -2.648 + 162.730, -6.253 \leq \sigma_\alpha^2 \leq 160.082$

25.19. a.  $e_{ij}$ :

$i$	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$
1	-.175	-1.300	.325	-2.050	3.200
2	.025	4.900	-3.475	1.150	-2.600
3	-.575	-1.700	.925	-1.450	2.800
4	.025	1.900	-1.475	2.150	-2.600
5	.025	-1.100	.525	-.850	1.400
6	.025	-1.100	2.525	-.850	-.600
7	-.175	-.300	-.675	2.950	-1.800
8	.825	-1.300	1.325	-1.050	.200

$$r = .985$$

c.  $H_0: D = 0, H_a: D \neq 0. SSBL.TR^* = 27.729, SSRem^* = 94.521,$

$$F^* = (27.729/1) \div (94.521/27) = 7.921, F(.995; 1, 27) = 9.34.$$

If  $F^* \leq 9.34$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

25.20. a.

Source	$SS$	$df$	$MS$
Blocks	4,826.375	7	689.48214
Paint type	531.350	4	132.83750
Error	122.250	28	4.36607
Total	5,479.975	39	

b.  $H_0$ : all  $\tau_j$  equal zero ( $j = 1, \dots, 5$ ),  $H_a$ : not all  $\tau_j$  equal zero.

$$F^* = 132.83750/4.36607 = 30.425, F(.95; 4, 28) = 2.71.$$

If  $F^* \leq 2.71$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

c.  $\bar{Y}_1 = 20.500, \bar{Y}_2 = 23.625, \bar{Y}_3 = 19.000, \bar{Y}_4 = 29.375, \bar{Y}_5 = 21.125, \hat{L}_1 = \bar{Y}_1 - \bar{Y}_2 = -3.125, \hat{L}_2 = \bar{Y}_1 - \bar{Y}_3 = 1.500, \hat{L}_3 = \bar{Y}_1 - \bar{Y}_4 = -8.875, \hat{L}_4 = \bar{Y}_1 - \bar{Y}_5 = -.625, s\{\hat{L}_i\} = 1.0448 (i = 1, \dots, 4), B = t(.9875; 28) = 2.369$

$$-3.125 \pm 2.369(1.0448) \quad -5.60 \leq L_1 \leq -.65$$

$$1.500 \pm 2.369(1.0448) \quad -.98 \leq L_2 \leq 3.98$$

$$-8.875 \pm 2.369(1.0448) \quad -11.35 \leq L_3 \leq -6.40$$

$$-.625 \pm 2.369(1.0448) \quad -3.10 \leq L_4 \leq 1.85$$

d.  $\hat{L} = \frac{1}{3}(\bar{Y}_1 + \bar{Y}_3 + \bar{Y}_5) - \frac{1}{2}(\bar{Y}_2 + \bar{Y}_4) = -6.29167, s\{\hat{L}\} = .6744, t(.975; 28) = 2.048, -6.29167 \pm 2.048(.6744), -7.67 \leq L \leq -4.91$

25.23. a.  $H_0: \sigma_{\alpha\beta\gamma}^2 = 0, H_a: \sigma_{\alpha\beta\gamma}^2 > 0. F^* = MSABC/MSE = 1.49/2.30 = .648, F(.975; 8, 60) = 2.41. If F^* \leq 2.41 conclude H_0, otherwise H_a. Conclude H_0. P\text{-value}=.27.$

b.  $H_0: \sigma_{\alpha\beta}^2 = 0, H_a: \sigma_{\alpha\beta}^2 > 0. F^* = MSAB/MSABC = 2.40/1.49 = 1.611, F(.99; 2, 8) = 8.65. If F^* \leq 8.65 conclude H_0, otherwise H_a. Conclude H_0.$

c.  $H_0: \sigma_{\beta}^2 = 0, H_a: \sigma_{\beta}^2 > 0. F^{**} = MSB/(MSAB + MSBC - MSABC) = 4.20/(2.40 + 3.13 - 1.49) = 1.04, df = 16.32161/5.6067 = 2.91, F(.99; 1, 3) = 34.1. If F^{**} \leq 34.1 conclude H_0, otherwise H_a. Conclude H_0.$

d.  $s_{\alpha}^2 = (MSA - MSAB - MSAC + MSABC)/nbc = .126,$

$$df = [(8.650/30) - (2.40/30) - (3.96/30) + (1.49/30)]^2$$

$$\div \left[ \frac{(8.65/30)^2}{2} + \frac{(2.40/30)^2}{2} + \frac{(3.96/30)^2}{8} + \frac{(1.49/30)^2}{8} \right] = .336$$

$$\chi^2(.025; 1) = .001, \chi^2(.975; 1) = 5.02$$

$$.008 = \frac{.336(.126)}{5.02} \leq \sigma_{\alpha}^2 \leq \frac{.336(.126)}{.001} = 42.336$$

25.26. a.  $\hat{\mu}_{..} = 55.593, \hat{\beta}_1 = .641, \hat{\beta}_2 = .218, \hat{\sigma}_{\alpha}^2 = 5.222, \hat{\sigma}_{\alpha\beta}^2 = 15.666, \hat{\sigma}^2 = 55.265, \text{no}$   
(Note: Unrestricted estimators are same except that variance component for random effect A is zero.)

b. Estimates remain the same.

c.  $H_0: \sigma_{\alpha\beta}^2 = 0, H_a: \sigma_{\alpha\beta}^2 > 0. z(.99) = 2.326, s\{\hat{\sigma}_{\alpha\beta}^2\} = 13.333, z^* = 15.666/13.333 = 1.175. If z^* \leq 2.326 conclude H_0, otherwise H_a. Conclude H_0. P\text{-value} = .12.$

d.  $H_0: \beta_1 = \beta_2 = \beta_3 = 0, H_a: \text{not all } \beta_j = 0 (j = 1, 2, 3). -2\log_e L(R) = 295.385, -2\log_e L(F) = 295.253, X^2 = 295.385 - 295.253 = .132, \chi^2(.99; 2) = 9.21. If X^2 \leq 9.21 conclude H_0, otherwise H_a. Conclude H_0. P\text{-value} = .94.$

e.  $z(.995) = 2.576, 15.666 \pm 2.576(13.333), -18.680 \leq \alpha_{\alpha\beta}^2 \leq 50.012$



# Chapter 26

## NESTED DESIGNS, SUBSAMPLING, AND PARTIALLY NESTED DESIGNS

26.9. a.  $e_{ijk}$ :

$i = 1$				$i = 2$			
$k$	$j = 1$	$j = 2$	$j = 3$	$k$	$j = 1$	$j = 2$	$j = 3$
1	1.8	-12.8	-9.6	1	-7.2	-2.6	8.8
2	15.8	-.8	7.4	2	3.8	-15.6	-8.2
3	-5.2	3.2	16.4	3	-15.2	6.4	-10.2
4	-.2	-3.8	-14.6	4	7.8	11.4	11.8
5	-12.2	14.2	.4	5	10.8	.4	-2.2

  

$i = 3$			
$k$	$j = 1$	$j = 2$	$j = 3$
1	-5.8	-9.8	-12.0
2	11.2	12.2	0.0
3	-.8	-.8	17.0
4	-12.8	3.2	2.0
5	8.2	-4.8	-7.0

$r = .987$

26.10. a.

Source	$SS$	$df$	$MS$
States ( $A$ )	6,976.84	2	3,488.422
Cities within states [ $B(A)$ ]	167.60	6	27.933
Error ( $E$ )	3,893.20	36	108.144
Total	11,037.64	44	

- b.  $H_0$ : all  $\alpha_i$  equal zero ( $i = 1, 2, 3$ ),  $H_a$ : not all  $\alpha_i$  equal zero.  $F^* = 3,488.422/108.144 = 32.257$ ,  $F(.95; 2, 36) = 3.26$ . If  $F^* \leq 3.26$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+
- c.  $H_0$ : all  $\beta_{j(i)}$  equal zero,  $H_a$ : not all  $\beta_{j(i)}$  equal zero.  $F^* = 27.933/108.144 = .258$ ,  $F(.95; 6, 36) = 2.36$ . If  $F^* \leq 2.36$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

$P$ -value = .95

d.  $\alpha \leq .10$

26.11. a.  $\bar{Y}_{11.} = 40.2$ ,  $s\{\bar{Y}_{11.}\} = 4.6507$ ,  $t(.975; 36) = 2.0281$ ,

$40.2 \pm 2.0281(4.6507)$ ,  $30.77 \leq \mu_{11} \leq 49.63$

b.  $\bar{Y}_{1..} = 40.8667$ ,  $\bar{Y}_{2..} = 57.3333$ ,  $\bar{Y}_{3..} = 26.8667$ ,  $s\{\bar{Y}_{i..}\} = 2.6851$  ( $i = 1, 2, 3$ ),  
 $t(.995; 36) = 2.7195$

$40.8667 \pm 2.7195(2.6851)$        $33.565 \leq \mu_{1.} \leq 48.169$

$57.3333 \pm 2.7195(2.6851)$        $50.031 \leq \mu_{2.} \leq 64.635$

$26.8667 \pm 2.7195(2.6851)$        $19.565 \leq \mu_{3.} \leq 34.169$

c.  $\hat{L}_1 = \bar{Y}_{1..} - \bar{Y}_{2..} = -16.4666$ ,  $\hat{L}_2 = \bar{Y}_{1..} - \bar{Y}_{3..} = 14.0000$ ,  $\hat{L}_3 = \bar{Y}_{2..} - \bar{Y}_{3..} = 30.4666$ ,  
 $s\{\hat{L}_i\} = 3.7973$  ( $i = 1, 2, 3$ ),  $q(.90; 3, 36) = 2.998$ ,  $T = 2.120$

$-16.4666 \pm 2.120(3.7973)$        $-24.52 \leq L_1 \leq -8.42$

$14.0000 \pm 2.120(3.7973)$        $5.95 \leq L_2 \leq 22.05$

$30.4666 \pm 2.120(3.7973)$        $22.42 \leq L_3 \leq 38.52$

d.  $\hat{L} = 12.4$ ,  $s\{\hat{L}\} = 6.5771$ ,  $t(.975; 36) = 2.0281$ ,  $12.4 \pm 2.0281(6.5771)$ ,  $-.94 \leq L \leq 25.74$

26.12. a.  $\beta_{j(i)}$  are independent  $N(0, \sigma_\beta^2)$ ;  $\beta_{j(i)}$  are independent of  $\epsilon_{k(j)}$ .

b.  $\hat{\sigma}_\beta^2 = 0$ , yes.

c.  $H_0: \sigma_\beta^2 = 0$ ,  $H_a: \sigma_\beta^2 > 0$ .  $F^* = 27.933/108.144 = .258$ ,  $F(.90; 6, 36) = 1.94$ .

If  $F^* \leq 1.94$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .95

d.  $H_0$ : all  $\alpha_i$  equal zero ( $i = 1, 2, 3$ ),  $H_a$ : not all  $\alpha_i$  equal zero.  $F^* = 3,488.422/27.933 = 124.885$ ,  $F(.90; 2, 6) = 3.46$ . If  $F^* \leq 3.46$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

e. See Problem 26.11c.  $s\{\hat{L}_i\} = 1.9299$  ( $i = 1, 2, 3$ ),  $q(.90; 3, 6) = 3.56$ ,  $T = 2.5173$

$-16.4666 \pm 2.5173(1.9299)$        $-21.32 \leq L_1 \leq -11.61$

$14.0000 \pm 2.5173(1.9299)$        $9.14 \leq L_2 \leq 18.86$

$30.4666 \pm 2.5173(1.9299)$        $25.61 \leq L_3 \leq 35.32$

f.  $H_0$ : all  $\sigma^2\{\beta_{j(i)}\}$  are equal ( $i = 1, 2, 3$ ),  $H_a$ : not all  $\sigma^2\{\beta_{j(i)}\}$  are equal.

$H^* = 37.27/16.07 = 2.32$ ,  $H(.95; 3, 2) = 87.5$ .

If  $H^* \leq 87.5$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

26.13. a.  $\alpha_i$  are independent  $N(0, \sigma_\alpha^2)$ ;  $\beta_{j(i)}$  are independent  $N(0, \sigma_\beta^2)$ ;

$\alpha_i$ ,  $\beta_{j(i)}$ , and  $\epsilon_{k(ij)}$  are independent.

b.  $\hat{\sigma}_\beta^2 = 0$ ,  $\hat{\sigma}_\alpha^2 = 230.699$

c.  $H_0: \sigma_\alpha^2 = 0$ ,  $H_a: \sigma_\alpha^2 > 0$ .  $F^* = 3,488.422/27.933 = 124.885$ ,  $F(.99; 2, 6) = 10.9$ .

If  $F^* \leq 10.9$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

- d.  $c_1 = 1/15, c_2 = -1/15, MS_1 = 3488.422, MS_2 = 27.933, df_1 = 2, df_2 = 6,$   
 $F_1 = F(.995; 2, \infty) = 5.30, F_2 = F(.995; 6, \infty) = 3.09, F_3 = F(.995; \infty, 2) = 200,$   
 $F_4 = F(.995; \infty, 6) = 8.88, F_5 = F(.995; 2, 6) = 14.5, F_6 = F(.995; 6, 2) = 199,$   
 $G_1 = .8113, G_2 = .6764, G_3 = -1.2574, G_4 = -93.0375, H_L = 187.803, H_U =$   
 $46, 279.30, 230.699 - 187.803, 230.699 + 46, 279.30, 42.90 \leq \sigma_\alpha^2 \leq 46, 510.00$
- e.  $\bar{Y}_{..} = 41.6889, s\{\bar{Y}_{..}\} = 8.8046, t(.995; 2) = 9.925, 41.6889 \pm 9.925(8.8046),$   
 $-45.70 \leq \mu_{..} \leq 129.07$

26.19.  $e_{ijk}$ :

				$i = 1$								$i = 2$			
		$k$		$j = 1$	$j = 2$	$j = 3$					$k$	$j = 1$	$j = 2$	$j = 3$	
1		-.4000	.0333	-.3667					1	.0667	.4333	-.2000			
2		.0000	.3333	.0333					2	-.2333	.0667	.3000			
3		.4000	-.3667	.3333					3	.1667	-.3667	-.1000			
				$i = 3$								$i = 4$			
		$k$		$j = 1$	$j = 2$	$j = 3$					$k$	$j = 1$	$j = 2$	$j = 3$	
1		-.4333	-.1333	-.3667					1	-.0667	-.3000	.4000			
2		.1667	.4667	.3333					2	.4333	.2000	.0000			
3		.2667	-.3333	-.0667					3	-.3667	.1000	-.4000			

$r = .972$

26.20. a.

Source	SS	df	MS
Plants	343.1789	3	114.3930
Leaves, within plants	187.4533	8	23.4317
Observations, within leaves	3.0333	24	.1264
Total	533.6655	35	

- b.  $H_0: \sigma_\tau^2 = 0, H_a: \sigma_\tau^2 > 0. F^* = 114.3930/23.4317 = 4.88, F(.95; 3, 8) = 4.07.$   
 If  $F^* \leq 4.07$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = .03
- c.  $H_0: \sigma^2 = 0, H_a: \sigma^2 > 0. F^* = 23.4317/.1264 = 185.38, F(.95; 8, 24) = 2.36.$   
 If  $F^* \leq 2.36$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+
- d.  $\bar{Y}_{..} = 14.26111, s\{\bar{Y}_{..}\} = 1.7826, t(.975; 3) = 3.182,$   
 $14.26111 \pm 3.182(1.7826), 8.59 \leq \mu_{..} \leq 19.93$
- e.  $\hat{\sigma}_\tau^2 = 10.1068, \hat{\sigma}^2 = 7.7684, \hat{\sigma}_\eta^2 = .1264$
- f.  $c_1 = 1/9 = .1111, c_2 = -1/9 = -.1111, MS_1 = 114.3930, MS_2 = 23.4317,$   
 $df_1 = 3, df_2 = 8, F_1 = F(.95; 3, \infty) = 2.60, F_2 = F(.95; 8, \infty) = 1.94, F_3 =$   
 $F(.95; \infty, 3) = 8.53, F_4 = F(.95; \infty, 8) = 2.93, F_5 = F(.95; 3, 8) = 4.07, F_6 =$   
 $F(.95; 8, 3) = 8.85, G_1 = .6154, G_2 = .4845, G_3 = -.1409, G_4 = -1.5134,$   
 $H_L = 9.042, H_U = 95.444, 10.1068 - 9.042, 10.1068 + 95.444, 1.065 \leq \sigma_\tau^2 \leq 105.551$



# Chapter 27

## REPEATED MEASURES AND RELATED DESIGNS

27.6. a.  $e_{ij}$ :

$i$	$j = 1$	$j = 2$	$j = 3$
1	-1.2792	-.2417	1.5208
2	-.8458	.6917	.1542
3	.6208	.0583	-.6792
4	.5542	.1917	-.7458
5	.5208	-.3417	-.1792
6	-.1458	.3917	-.2458
7	.9875	-.7750	-.2125
8	-.4125	.0250	.3875

$r = .992$

d.  $H_0: D = 0$ ,  $H_a: D \neq 0$ .  $SSTR.S = 9.5725$ ,  $SSTR.S^* = 2.9410$ ,  $SSRem^* = 6.6315$ ,  $F^* = (2.9410/1) \div (6.6315/13) = 5.765$ ,  $F(.99; 1, 13) = 9.07$ . If  $F^* \leq 9.07$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .032

27.7. a.

Source	$SS$	$df$	$MS$
Stores	745.1850	7	106.4550
Prices	67.4808	2	33.7404
Error	9.5725	14	.68375
Total	822.2383	23	

b.  $H_0$ : all  $\tau_j$  equal zero ( $j = 1, 2, 3$ ),  $H_a$ : not all  $\tau_j$  equal zero.  $F^* = 33.7404/.68375 = 49.346$ ,  $F(.95; 2, 14) = 3.739$ . If  $F^* \leq 3.739$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

c.  $\bar{Y}_1 = 55.4375$ ,  $\bar{Y}_2 = 53.6000$ ,  $\bar{Y}_3 = 51.3375$ ,  $\hat{L}_1 = \bar{Y}_1 - \bar{Y}_2 = 1.8375$ ,  $\hat{L}_2 = \bar{Y}_1 - \bar{Y}_3 = 4.1000$ ,  $\hat{L}_3 = \bar{Y}_2 - \bar{Y}_3 = 2.2625$ ,  $s\{\hat{L}_i\} = .413446$  ( $i = 1, 2, 3$ ),  $q(.95; 3, 14) = 3.70$ ,  $T = 2.616$

$$1.8375 \pm 2.616(.413446) \quad .756 \leq L_1 \leq 2.919$$

$$4.1000 \pm 2.616(.413446) \quad 3.018 \leq L_2 \leq 5.182$$

$$2.2625 \pm 2.616(.413446) \quad 1.181 \leq L_3 \leq 3.344$$



d.  $\hat{E} = 48.08$

27.9.  $H_0$ : all  $\tau_j$  equal zero ( $j = 1, 2, 3$ ),  $H_a$ : not all  $\tau_j$  equal zero.  $MSTR = 8$ ,  $MSTR.S = 0$ ,  $F_R^* = 8/0$ . Note: Nonparametric  $F$  test results in  $SSTR.S = 0$  and therefore should not be used.

27.13. a.  $e_{ijk}$ :

		$k = 1$	$k = 2$	$k = 3$	$k = 4$
$j = 1$	$i = 1$	9.250	-8.750	1.250	-1.750
	$i = 2$	-11.750	-2.750	15.250	-.750
	$i = 3$	7.750	-5.250	5.750	-8.250
	$i = 4$	-5.250	16.750	-22.250	10.750
$j = 2$	$i = 1$	3.625	-3.125	-13.875	13.375
	$i = 2$	15.375	6.625	7.875	-29.875
	$i = 3$	-8.375	-3.125	-3.875	15.375
	$i = 4$	-10.625	-.375	9.875	1.125

$r = .981$

27.14. a.  $H_0: \sigma^2\{\rho_{i(1)}\} = \sigma^2\{\rho_{i(2)}\}$ ,  $H_a: \sigma^2\{\rho_{i(1)}\} \neq \sigma^2\{\rho_{i(2)}\}$ .  
 $SSS(A_1) = 1,478,757.00$ ,  $SSS(A_2) = 1,525,262.25$ ,  
 $H^* = (1,525,262.25/3) \div (1,478,757.00/3) = 1.03$ ,  $H(.99; 2, 3) = 47.5$ .  
 If  $H^* \leq 47.5$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

b.  $H_0: \sigma^2\{\epsilon_{1jk}\} = \sigma^2\{\epsilon_{2jk}\}$ ,  $H_a: \sigma^2\{\epsilon_{1jk}\} \neq \sigma^2\{\epsilon_{2jk}\}$ .  
 $SSB.S(A_1) = 1,653.00$ ,  $SSB.S(A_2) = 2,172.25$ ,  
 $H^* = (2,172.25/9) \div (1,653.00/9) = 1.31$ ,  $H(.99; 2, 9) = 6.54$ .  
 If  $H^* \leq 6.54$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

27.15. a.

Source	$SS$	$df$	$MS$
$A$ (type display)	266,085.1250	1	266,085.1250
$S(A)$	3,004,019.2500	6	500,669.8750
$B$ (time)	53,321.6250	3	17,773.8750
$AB$ interactions	690.6250	3	230.2083
Error	3,825.2500	18	212.5139
Total	3,327,941.8750	31	

b.  $\bar{Y}_{.11} = 681.500$ ,  $\bar{Y}_{.12} = 696.500$ ,  $\bar{Y}_{.13} = 671.500$ ,  $\bar{Y}_{.14} = 785.500$ ,  
 $\bar{Y}_{.21} = 508.500$ ,  $\bar{Y}_{.22} = 512.250$ ,  $\bar{Y}_{.23} = 496.000$ ,  $\bar{Y}_{.24} = 588.750$

c.  $H_0$ : all  $(\alpha\beta)_{jk}$  equal zero,  $H_a$ : not all  $(\alpha\beta)_{jk}$  equal zero.  
 $F^* = 230.2083/212.5139 = 1.08$ ,  $F(.975; 3, 18) = 3.95$ .

If  $F^* \leq 3.95$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .38

d.  $H_0: \alpha_1 = \alpha_2 = 0$ ,  $H_a$ : not both  $\alpha_j$  equal zero.

$F^* = 266,085.1250/500,669.8750 = .53$ ,  $F(.975; 1, 6) = 8.81$ .

If  $F^* \leq 8.81$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .49

$H_0$ : all  $\beta_k$  equal zero ( $k = 1, \dots, 4$ ),  $H_a$ : not all  $\beta_k$  equal zero.

$F^* = 17,773.8750/212.5139 = 83.636$ ,  $F(.975; 3, 18) = 3.95$ .

If  $F^* \leq 3.95$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = 0+

- e.  $\bar{Y}_{.1} = 708.750$ ,  $\bar{Y}_{.2} = 526.375$ ,  $\bar{Y}_{.1} = 595.000$ ,  $\bar{Y}_{.2} = 604.375$ ,  $\bar{Y}_{.3} = 583.750$ ,  
 $\bar{Y}_{.4} = 687.125$ ,  $\hat{L}_1 = 182.375$ ,  $\hat{L}_2 = -9.375$ ,  $\hat{L}_3 = 20.625$ ,  $\hat{L}_4 = -103.375$ ,  
 $s\{\hat{L}_1\} = 250.1674$ ,  $s\{\hat{L}_i\} = 7.2889$  ( $i = 2, 3, 4$ ),  $B_1 = t(.9875; 6) = 2.969$ ,  $B_i =$   
 $t(.9875; 18) = 2.445$  ( $i = 2, 3, 4$ )

$$\begin{array}{ll} 182.375 \pm 2.969(250.1674) & -560.372 \leq L_1 \leq 925.122 \\ -9.375 \pm 2.445(7.2889) & -27.196 \leq L_2 \leq 8.446 \\ 20.625 \pm 2.445(7.2889) & 2.804 \leq L_3 \leq 38.446 \\ -103.375 \pm 2.445(7.2889) & -121.196 \leq L_4 \leq -85.554 \end{array}$$

27.18. a.  $e_{ijk}$ :

$i$	$j = 1$		$j = 2$	
	$k = 1$	$k = 2$	$k = 1$	$k = 2$
1	-.045	.045	.045	-.045
2	-.120	.120	.120	-.120
3	.080	-.080	-.080	.080
4	-.045	.045	.045	-.045
5	.080	-.080	-.080	.080
6	.055	-.055	-.055	.055
7	.030	-.030	-.030	.030
8	-.045	.045	.045	-.045
9	.055	-.055	-.055	.055
10	-.045	.045	.045	-.045

$r = .973$

27.19. a.

Source	$SS$	$df$	$MS$
Subjects	154.579	9	17.175
$A$	3.025	1	3.025
$B$	14.449	1	11.449
$AB$	.001	1	.001
$AS$	2.035	9	.226
$BS$	5.061	9	.562
$ABS$	.169	9	.019
Total	176.319	39	

- b.  $\bar{Y}_{.11} = 3.93$ ,  $\bar{Y}_{.12} = 5.01$ ,  $\bar{Y}_{.21} = 4.49$ ,  $\bar{Y}_{.22} = 5.55$

- c.  $H_0$ : all  $(\alpha\beta)_{jk}$  equal zero,  $H_a$ : not all  $(\alpha\beta)_{jk}$  equal zero.

$F^* = .001/.019 = .05$ ,  $F(.995; 1, 9) = 13.6$ .

If  $F^* \leq 13.6$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .82

d.  $H_0: \alpha_1 = \alpha_2 = 0$ ,  $H_a$ : not both  $\alpha_j$  equal zero.

$$F^* = 3.025/.226 = 13.38, F(.995; 1, 9) = 13.6.$$

If  $F^* \leq 13.6$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .  $P$ -value = .005

$H_0: \beta_1 = \beta_2 = 0$ ,  $H_a$ : not both  $\beta_k$  equal zero.

$$F^* = 11.449/.562 = 20.36, F(.995; 1, 9) = 13.6.$$

If  $F^* \leq 13.6$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = .001

e.  $\hat{L}_1 = .56$ ,  $\hat{L}_2 = 1.08$ ,  $\hat{L}_3 = -.52$ ,  $\hat{L}_4 = 1.62$ ,

$$s\{\hat{L}_i\} = .0613 \quad (i = 1, \dots, 4), B = t(.99375; 27) = 3.11$$

$.56 \pm 3.11(.0613)$	$.37 \leq L_1 \leq .75$
$1.08 \pm 3.11(.0613)$	$.89 \leq L_2 \leq 1.27$
$-.52 \pm 3.11(.0613)$	$-.71 \leq L_3 \leq -.33$
$1.62 \pm 3.11(.0613)$	$1.43 \leq L_4 \leq 1.81$

# Chapter 28

## BALANCED INCOMPLETE BLOCK, LATIN SQUARE, AND RELATED DESIGNS

28.8.  $e_{ij}$ :

$i$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
1	13.2083	8.8333	-22.0417	
2	-7.9167	4.7083		3.2083
3	-5.2917		-1.5417	6.8333
4		-13.5417	23.5833	-10.0417

$r = .996$

28.9. a.  $\hat{\mu}_{.} = 297.667$ ,  $\hat{\tau}_1 = -45.375$ ,  $\hat{\tau}_2 = -41.000$ ,  $\hat{\tau}_3 = 30.875$ ,  $\hat{\tau}_4 = 55.550$

$\hat{\mu}_{.1} = 252.292$ ,  $\hat{\mu}_{.2} = 256.667$ ,  $\hat{\mu}_{.3} = 328.542$ ,  $\hat{\mu}_{.4} = 353.167$

b.  $H_0: \tau_1 = \tau_2 = \tau_3 = 0$ ,  $H_a$ : not all  $\tau_j$  equal zero.  $SSE(F) = 1750.9$ ,  $SSE(R) = 22480$ ,  $F^* = (20729.1/3) \div (1750.9/5) = 19.73$ ,  $F(.95; 3, 5) = 5.41$ . If  $F^* \leq 5.41$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = .003

c.  $H_0: \rho_1 = \rho_2 = \rho_3 = 0$ ,  $H_a$ : not all  $\rho_i$  equal zero.  $SSE(F) = 14.519$ ,  $SSE(R) = 22789$ ,  $F^* = (21038.1/3) \div (1750.9/5) = 20.03$ ,  $F(.95; 3, 5) = 5.41$ . If  $F^* \leq 5.41$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = .003

d.  $\hat{\mu}_{.1} = 252.292$ ,  $s^2(\hat{\mu}_{.1}) = s^2(\hat{\mu}_{.}) + s^2(\hat{\tau}_1) = (.08333 + .28125)350.2 = 127.68$ ,  $B = t(.975; 5) = 2.571$ ,  $252.292 \pm 2.571(11.30)$ ,  $223.240 \leq \mu_{.1} \leq 281.344$

e.

95% C.I.	lower	center	upper
$\mu_{.1} - \mu_{.2}$	-64.19	-4.375	55.44
$\mu_{.1} - \mu_{.3}$	-136.07	-76.250	-16.43
$\mu_{.1} - \mu_{.4}$	-160.69	-100.875	-41.06
$\mu_{.2} - \mu_{.3}$	-131.70	-71.87	-12.06
$\mu_{.2} - \mu_{.4}$	-156.30	-96.50	-36.68
$\mu_{.3} - \mu_{.4}$	-84.44	-24.63	35.19

28.10.  $r = 4$ , and  $r_b = 3$ ,  $df_e = 4n - 4 - 4n/3 + 1 = 8n/3 - 3$ .

Since  $n_p = n(3 - 1)/(4 - 1) = 2n/3$ ,  $\sigma^2\{\hat{D}_j\} = 2\sigma^2(3)/(4n_p) = 9\sigma^2/(4n)$

$$T\sigma\{\hat{D}_j\} = \frac{1}{\sqrt{2}}q[.95; 4, 8n/3 - 3]\sqrt{\frac{9\sigma^2}{4n}}$$

For  $\sigma^2 = 2.0$  and  $T\sigma\{\hat{D}_j\} \leq 1.5$ , so we need to iterate to find  $n$  so that

$$n \geq 2q^2[.95; 4, 8n/3 - 3]$$

We iteratively find  $n \geq 28$ . Since design 2 in Table 28.1 has  $n = 3$ , we require that design 2 be repeated 10 times. Thus,  $n = 30$ , and  $n_b = 40$ .

28.14.  $e_{ijk}$ :

$i$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
1	-.1375	.0875	-.0125	.0625
2	-.0125	-.0125	.1625	-.1375
3	.1375	-.0875	-.0625	.0125
4	.0125	.0125	-.0875	.0625

$$r = .986$$

28.15. a.  $\bar{Y}_{..1} = 1.725$ ,  $\bar{Y}_{..2} = 1.900$ ,  $\bar{Y}_{..3} = 2.175$ ,  $\bar{Y}_{..4} = 2.425$

b.

Source	$SS$	$df$	$MS$
Rows (sales volumes)	5.98187	3	1.99396
Columns (locations)	.12188	3	.04062
Treatments (prices)	1.13688	3	.37896
Error	.11875	6	.01979
Total	7.35938	15	

$H_0$ : all  $\tau_k$  equal zero ( $k = 1, \dots, 4$ ),  $H_a$ : not all  $\tau_k$  equal zero.  $F^* = .37896/.01979 = 19.149$ ,  $F(.95; 3, 6) = 4.76$ . If  $F^* \leq 4.76$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .  $P$ -value = .002

c.  $\hat{L}_1 = \bar{Y}_{..1} - \bar{Y}_{..2} = -.175$ ,  $\hat{L}_2 = \bar{Y}_{..1} - \bar{Y}_{..3} = -.450$ ,  $\hat{L}_3 = \bar{Y}_{..1} - \bar{Y}_{..4} = -.700$ ,  
 $\hat{L}_4 = \bar{Y}_{..2} - \bar{Y}_{..3} = -.275$ ,  $\hat{L}_5 = \bar{Y}_{..2} - \bar{Y}_{..4} = -.525$ ,  $\hat{L}_6 = \bar{Y}_{..3} - \bar{Y}_{..4} = -.250$ ,  
 $s\{\hat{L}_i\} = .09947$  ( $i = 1, \dots, 6$ ),  $q(.90; 4, 6) = 4.07$ ,  $T = 2.8779$

$$\begin{aligned} &-.175 \pm 2.8779(.09947) & &-.461 \leq L_1 \leq .111 \\ &-.450 \pm 2.8779(.09947) & &-.736 \leq L_2 \leq -.164 \\ &-.700 \pm 2.8779(.09947) & &-.986 \leq L_3 \leq -.414 \\ &-.275 \pm 2.8779(.09947) & &-.561 \leq L_4 \leq .011 \\ &-.525 \pm 2.8779(.09947) & &-.811 \leq L_5 \leq -.239 \\ &-.250 \pm 2.8779(.09947) & &-.536 \leq L_6 \leq .036 \end{aligned}$$

28.16. a.  $\hat{E}_1 = 21.1617$ ,  $\hat{E}_2 = 1.2631$ ,  $\hat{E}_3 = 25.9390$

28.20.  $\phi = 3.399$ ,  $1 - \beta \cong .99$

28.24. a.  $Y_{ijk} = \mu_{...} + \rho_1 X_{ijk1} + \rho_2 X_{ijk2} + \rho_3 X_{ijk3} + \kappa_1 X_{ijk4} + \kappa_2 X_{ijk5}$   
 $+ \kappa_3 X_{ijk6} + \tau_1 X_{ijk7} + \tau_2 X_{ijk8} + \tau_3 X_{ijk9} + \epsilon_{(ijk)}$

$$X_{ijk1} = \begin{cases} 1 & \text{if experimental unit from row blocking class 1} \\ -1 & \text{if experimental unit from row blocking class 4} \\ 0 & \text{otherwise} \end{cases}$$

$X_{ijk2}$  and  $X_{ijk3}$  are defined similarly

$$X_{ijk4} = \begin{cases} 1 & \text{if experimental unit from column blocking class 1} \\ -1 & \text{if experimental unit from column blocking class 4} \\ 0 & \text{otherwise} \end{cases}$$

$X_{ijk5}$  and  $X_{ijk6}$  are defined similarly

$$X_{ijk7} = \begin{cases} 1 & \text{if experimental unit received treatment 1} \\ -1 & \text{if experimental unit received treatment 4} \\ 0 & \text{otherwise} \end{cases}$$

$X_{ijk8}$  and  $X_{ijk9}$  are defined similarly

b. Full model:

$$\hat{Y} = 2.05625 - .70625X_1 - .45625X_2 + .34375X_3 + .14375X_4 \\ - .05625X_5 - .00625X_6 - .33125X_7 - .15625X_8 + .11875X_9$$

$$SSE(F) = .1188$$

Reduced model:

$$\hat{Y} = 2.05625 - .70625X_1 - .45625X_2 + .34375X_3 + .14375X_4 - .05625X_5 - .00625X_6$$

$$SSE(R) = 1.2556$$

$H_0$ : all  $\tau_k$  equal zero ( $k = 1, 2, 3$ ),  $H_a$ : not all  $\tau_k$  equal zero.  $F^* = (1.1368/3) \div (.1188/6) = 19.138$ ,  $F(.95; 3, 6) = 4.76$ . If  $F^* \leq 4.76$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .

c.  $\hat{L} = \hat{\tau}_3 - (-\hat{\tau}_1 - \hat{\tau}_2 - \hat{\tau}_3) = 2\hat{\tau}_3 + \hat{\tau}_1 + \hat{\tau}_2 = -.250$ ,  $s^2\{\hat{\tau}_i\} = .00371$  ( $i = 1, 2, 3$ ),  $s\{\hat{\tau}_1, \hat{\tau}_2\} = s\{\hat{\tau}_1, \hat{\tau}_3\} = s\{\hat{\tau}_2, \hat{\tau}_3\} = -.00124$ ,  $s\{\hat{L}\} = .09930$ ,  $t(.975; 6) = 2.447$ ,  $-.250 \pm 2.447(.09930)$ ,  $-.493 \leq L \leq -.007$

d. (i) Full model:

$$\hat{Y} = 2.02917 - .67917X_1 - .53750X_2 + .37083X_3 + .17083X_4 - .02917X_5 \\ - .08750X_6 - .30417X_7 - .23750X_8 + .14583X_9$$

$$SSE(F) = .0483$$

Reduced model:

$$\hat{Y} = 2.05556 - .70556X_1 - .45833X_2 + .34444X_3 + .14444X_4 - .05556X_5 - .00833X_6$$

$$SSE(R) = 1.2556$$

$H_0$ : all  $\tau_k$  equal zero ( $k = 1, 2, 3$ ),  $H_a$ : not all  $\tau_k$  equal zero.  $F^* = (1.2073/3) \div (.0483/5) = 41.66$ ,  $F(.95; 3, 5) = 5.41$ . If  $F^* \leq 5.41$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_a$ .

(ii)  $\hat{L} = \hat{\tau}_1 - \hat{\tau}_2 = -.06667$ ,  $s^2\{\hat{\tau}_1\} = .00191$ ,  $s^2\{\hat{\tau}_2\} = .00272$ ,  $s\{\hat{\tau}_1, \hat{\tau}_2\} = -.00091$ ,  $s\{\hat{L}\} = .0803$ ,  $t(.975; 5) = 2.571$ ,  $-.06667 \pm 2.571(.0803)$ ,  $-.273 \leq L \leq .140$



# Chapter 29

## EXPLORATORY EXPERIMENTS – TWO-LEVEL FACTORIAL AND FRACTIONAL FACTORIAL DESIGNS

29.3. a. Six factors, two levels, 64 trials

b. No

29.6. a.  $\sigma^2\{b_1\} = \sigma^2/n_T = 5^2/64 = .391$ . Yes, yes

b.  $z(.975) = 1.96$ ,  $n_T = [1.96(5)/(.5)]^2 = 384.16$ ,  $384.16/64 = 6$  replicates

29.7. a.  $Y_i = \beta_0 X_{i0} + \beta_1 X_{i1} + \cdots + \beta_5 X_{i5} + \beta_{12} X_{i12} + \cdots + \beta_{45} X_{i45} + \beta_{123} X_{i123}$   
 $+ \cdots + \beta_{345} X_{i345} + \beta_{1234} X_{i1234} + \cdots + \beta_{2345} X_{i2345} + \beta_{12345} X_{i12345} + \epsilon_i$

Coef.	$b_q$	Coef.	$b_q$	Coef.	$b_q$	Coef.	$b_q$
$b_0$	6.853	$b_{14}$	-.239	$b_{123}$	.070	$b_{245}$	.076
$b_1$	1.606	$b_{15}$	.611	$b_{124}$	.020	$b_{345}$	-.576
$b_2$	-.099	$b_{23}$	-.134	$b_{125}$	-.118	$b_{1234}$	.062
$b_3$	1.258	$b_{24}$	-.127	$b_{134}$	-.378	$b_{1235}$	.323
$b_4$	-1.151	$b_{25}$	-.045	$b_{135}$	-.138	$b_{1245}$	.357
$b_5$	-1.338	$b_{34}$	-.311	$b_{145}$	-.183	$b_{1345}$	-.122
$b_{12}$	-.033	$b_{35}$	.912	$b_{234}$	.233	$b_{2345}$	-.292
$b_{13}$	.455	$b_{45}$	-.198	$b_{235}$	.055	$b_{12345}$	.043

29.8. a.  $Y_i = \beta_0 X_{i0} + \beta_1 X_{i1} + \cdots + \beta_5 X_{i5} + \beta_{12} X_{i12} + \cdots + \beta_{45} X_{i45} + \epsilon_i$



Coef.	$b_q$	$P$ -value	Coef.	$b_q$	$P$ -value
$b_0$	6.853		$b_{14}$	-.239	.340
$b_1$	1.606	.000	$b_{15}$	.611	.023
$b_2$	-.099	.689	$b_{23}$	-.134	.589
$b_3$	1.258	.000	$b_{24}$	-.127	.610
$b_4$	-1.151	.000	$b_{25}$	-.045	.855
$b_5$	-1.338	.000	$b_{34}$	-.311	.219
$b_{12}$	-.033	.892	$b_{35}$	.912	.002
$b_{13}$	.455	.080	$b_{45}$	-.198	.426

- b.  $H_0$ : Normal,  $H_a$ : not normal.  $r = .983$ . If  $r \geq .9656$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .
- c.  $H_0: \beta_q = 0$ ,  $H_a: \beta_q \neq 0$ .  $s\{b_q\} = .2432$ . If  $P$ -value  $\geq .0034$  conclude  $H_0$ , otherwise  $H_a$ . Active effects (see part a):  $\beta_1, \beta_3, \beta_4, \beta_5, \beta_{35}$

29.15. Defining relation:  $0 = 123 = 245 = 1345$

Confounding scheme:

0	=	123	=	245	=	1345
1	=	23	=	1245	=	345
2	=	13	=	45	=	12345
3	=	12	=	2345	=	145
4	=	1234	=	25	=	135
5	=	1235	=	24	=	134
14	=	234	=	125	=	35
15	=	235	=	124	=	34

Resolution = III, no

29.18. a. Defining relation:  $0 = 1235 = 2346 = 1247 = 1456 = 3457 = 1367 = 2567$ , resolution = IV, no

- b. Omitting four-factor and higher-order interactions:

1	=	235	=	247	=	367	=	456
2	=	135	=	147	=	346	=	567
3	=	125	=	167	=	246	=	457
4	=	127	=	156	=	236	=	357
5	=	123	=	146	=	267	=	347
6	=	137	=	145	=	234	=	257
7	=	124	=	136	=	256	=	345
12	=	35	=	47				
13	=	25	=	67				
14	=	27	=	56				
15	=	23	=	46				
16	=	37	=	45				
17	=	24	=	36				
26	=	34	=	57				

- c.  $Y_i = \beta_0 X_{i0} + \beta_1 X_{i1} + \cdots + \beta_7 X_{i7} + \beta_{12} X_{i12} + \beta_{13} X_{i13} + \beta_{14} X_{i14}$

$$+\beta_{15}X_{i15} + \beta_{16}X_{i16} + \beta_{17}X_{i17} + \beta_{26}X_{i26} + \epsilon_i$$

Coef.	$b_q$	Coef.	$b_q$	Coef.	$b_q$
$b_0$	8.028	$b_5$	.724	$b_{14}$	-.316
$b_1$	.127	$b_6$	-.467	$b_{15}$	.318
$b_2$	.003	$b_7$	-.766	$b_{16}$	.117
$b_3$	.021	$b_{12}$	.354	$b_{17}$	.021
$b_4$	-2.077	$b_{13}$	-.066	$b_{26}$	-.182

- e.  $H_0: \beta_{12} = \dots = \beta_{17} = \beta_{26} = 0$ ,  $H_a$ : not all  $\beta_q = 0$ .  $F^* = (6.046/7) \div (.1958/1) = 4.41$ ,  $F(.99; 7, 1) = 5,928$ . If  $F^* \leq 5,928$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

29.19. a.

Coef.	$b_q$	$P$ -value	Coef.	$b_q$	$P$ -value
$b_0$	8.028		$b_4$	-2.077	.000
$b_1$	.127	.581	$b_5$	.724	.011
$b_2$	.003	.989	$b_6$	-.467	.067
$b_3$	.021	.928	$b_7$	-.766	.008

- b.  $H_0$ : Case  $i$  not an outlier,  $H_a$ : case  $i$  an outlier ( $i = 3, 14$ ).  $t_3 = 2.70$ ,  $t_{14} = -4.09$ ,  $t(.99844; 7) = 4.41$ . If  $|t_i| \leq 4.41$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$  for both cases.
- c.  $H_0$ : Normal,  $H_a$ : not normal.  $r = .938$ . If  $r \geq .929$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .
- d.  $H_0: \beta_q = 0$ ,  $H_a: \beta_q \neq 0$ .  $s\{b_q\} = .2208$ . If  $P$ -value  $\geq .02$  conclude  $H_0$ , otherwise  $H_a$ . Active effects (see part a):  $\beta_4, \beta_5, \beta_7$
- e. Set  $X_4 = -1$ ,  $X_5 = 1$ ,  $X_7 = -1$  to maximize extraction.

29.26. b. The seven block effects are confounded with the following interaction terms:  $\beta_{135}, \beta_{146}, \beta_{236}, \beta_{245}, \beta_{1234}, \beta_{1256}, \beta_{3456}$

No, no

c. 
$$Y_i = \beta_0 X_{i0} + \beta_1 X_{i1} + \dots + \beta_6 X_{i6} + \beta_{12} X_{i12} + \dots + \beta_{56} X_{i56} + \beta_{123} X_{i123}$$

$$+ \dots + \beta_{456} X_{i456} + \beta_{1235} X_{i1235} + \dots + \beta_{2456} X_{i2456} + \beta_{12345} X_{i12345}$$

$$+ \dots + \beta_{23456} X_{i23456} + \beta_{123456} X_{i123456} + \alpha_1 Z_{i1} + \dots + \alpha_7 Z_{i7} + \epsilon_i$$

where  $\alpha_1, \dots, \alpha_7$  are the block effects

Coef.	$b_q$	Coef.	$b_q$	Coef.	$b_q$	Coef.	$b_q$
$b_0$	63.922	$b_{34}$	.297	$b_{246}$	-.391	$b_{2356}$	.766
$b_1$	2.297	$b_{35}$	.266	$b_{256}$	.078	$b_{2456}$	.203
$b_2$	5.797	$b_{36}$	.984	$b_{345}$	-.672	$b_{12345}$	-.297
$b_3$	2.172	$b_{45}$	-.422	$b_{346}$	.734	$b_{12346}$	-.391
$b_4$	2.359	$b_{46}$	-.141	$b_{356}$	-.734	$b_{12356}$	-.734
$b_5$	2.828	$b_{56}$	.516	$b_{456}$	-.234	$b_{12456}$	-.422
$b_6$	2.922	$b_{123}$	.422	$b_{1235}$	.578	$b_{13456}$	-.109
$b_{12}$	.547	$b_{124}$	.172	$b_{1236}$	.922	$b_{23456}$	.203
$b_{13}$	-.266	$b_{125}$	1.391	$b_{1245}$	.453	$b_{123456}$	.016
$b_{14}$	-.203	$b_{126}$	.984	$b_{1246}$	.109	Block 1	-4.172
$b_{15}$	-.797	$b_{134}$	.297	$b_{1345}$	-.797	Block 2	-.422
$b_{16}$	-.141	$b_{136}$	-.641	$b_{1346}$	.547	Block 3	1.203
$b_{23}$	-.641	$b_{145}$	-.109	$b_{1356}$	-1.109	Block 4	6.703
$b_{24}$	-1.141	$b_{156}$	-.547	$b_{1456}$	-.109	Block 5	-.797
$b_{25}$	.891	$b_{234}$	.234	$b_{2345}$	.328	Block 6	-1.047
$b_{26}$	.047	$b_{235}$	.266	$b_{2346}$	-.578	Block 7	-9.547

29.27. a.

Coef.	$b_q$	$P$ -value	Coef.	$b_q$	$P$ -value
$b_0$	63.922		$b_{26}$	.047	.935
$b_1$	2.297	.000	$b_{34}$	.297	.607
$b_2$	5.797	.000	$b_{35}$	.266	.645
$b_3$	2.172	.001	$b_{36}$	.984	.094
$b_4$	2.359	.000	$b_{45}$	-.422	.466
$b_5$	2.828	.000	$b_{46}$	-.141	.807
$b_6$	2.922	.000	$b_{56}$	.516	.373
$b_{12}$	.547	.346	Block 1	-4.172	.009
$b_{13}$	-.266	.645	Block 2	-.422	.782
$b_{14}$	-.203	.725	Block 3	1.203	.432
$b_{15}$	-.797	.172	Block 4	6.703	.000
$b_{16}$	-.141	.807	Block 5	-.797	.602
$b_{23}$	-.641	.270	Block 6	-1.047	.494
$b_{24}$	-1.141	.054	Block 7	-9.547	.000
$b_{25}$	.891	.128			

- b.  $H_0$ : Normal,  $H_a$ : not normal.  $r = .989$ . If  $r \geq .9812$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .
- c.  $H_0: \beta_q = 0$ ,  $H_a: \beta_q \neq 0$ .  $s\{\hat{\alpha}_i\} = 1.513$  for block effects,  $s\{b_q\} = .5719$  for factor effects. If  $P$ -value  $\geq .01$  conclude  $H_0$ , otherwise  $H_a$ . Active effects (see part a): Block effects 1, 4, 7, all main effects

29.28. a. See Problem 29.27a for estimated factor and block effects. (These do not change with subset model.)

- b. Maximum team effectiveness is accomplished by setting each factor at its high level.

- c.  $\hat{Y}_h = 82.297$ ,  $s\{\text{pred}\} = 4.857$ ,  $t(.975; 50) = 2.009$ ,  $82.297 \pm 2.009(4.857)$ ,  $72.54 \leq Y_{h(\text{new})} \leq 92.05$

29.32. a.

$i$	1	2	3	4	5	6	7	8
$s_i^2$	.0164	.0173	.0804	.1100	.0010	.0079	.0953	.1134
$\log_e s_i^2$	-4.109	-4.058	-2.521	-2.207	-6.949	-4.838	-2.351	-2.176

- b.  $\widehat{\log_e s_i^2} = -3.651 + .331X_{i1} + 1.337X_{i2} - .427X_{i3} - .275X_{i4} - .209X_{i12} + .240X_{i13} + .477X_{i14}$ .

$X_2$  appears to be active.

- c.  $\hat{v}_i = .006819$  (for  $i = 1, 2, 5, 6$ )

$\hat{v}_i = .09887$  (for  $i = 3, 4, 7, 8$ )

- d.  $\hat{Y}_i = 7.5800 + .0772X_{i1}$

- e. From the location model:  $X_1 = +1$ ; from the dispersion model:  $X_2 = -1$

- f. From dispersion model:  $\hat{s}^2 = \exp[-3.651 + 1.337(-1)] = .006819$ ,  
and a 97.5% P.I. is  $[\exp(-6.16), \exp(-3.82)]$ , or  $(.00211, .0219)$ .

- g.  $\widehat{MSE} = .006819 + (8 - 7.659)^2 = .124$



# Chapter 30

## RESPONSE SURFACE METHODOLOGY

30.11. b.

Coef.	$b_q$	$P$ -value	Coef.	$b_q$	$P$ -value
$b_0$	1.868		$b_{13}$	-.038	.471
$b_1$	.190	.007	$b_{23}$	-.062	.251
$b_2$	.195	.006	$b_{11}$	.228	.044
$b_3$	-.120	.039	$b_{22}$	-.047	.602
$b_{12}$	.162	.020	$b_{33}$	.028	.757

- d.  $H_0: \beta_q = 0$ ,  $H_a: \beta_q \neq 0$ .  $s\{b_q\} = .0431$  (for linear effects),  $s\{b_q\} = .0481$  (for interaction effects),  $s\{b_q\} = .0849$  (for quadratic effects). If  $P$ -value  $\geq .05$  conclude  $H_0$ , otherwise  $H_a$ . Active effects (see part b):  $\beta_1, \beta_2, \beta_3, \beta_{12}, \beta_{11}$

30.12. a.

Coef.	$b_q$	Coef.	$b_q$
$b_0$	1.860	$b_3$	-.120
$b_1$	.190	$b_{12}$	.162
$b_2$	.195	$b_{11}$	.220

- b.  $H_0$ : Normal,  $H_a$ : not normal.  $r = .947$ . If  $r \geq .938$  conclude  $H_0$ , otherwise  $H_a$ . Conclude  $H_0$ .

30.14. a.

Design Matrix:

$X_1$	$X_2$
-.707	-.707
.707	-.707
-.707	.707
.707	.707
-1	0
1	0
0	-1
0	1
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0

Corner Points:

$X_1$	$X_2$
-.707	-.707
.707	-.707
-.707	.707
.707	.707

b.

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} .125 & 0 & 0 & -.125 & -.125 & 0 \\ 0 & .250 & 0 & 0 & 0 & 0 \\ 0 & 0 & .250 & 0 & 0 & 0 \\ -.125 & 0 & 0 & .5 & 0 & 0 \\ -.125 & 0 & 0 & 0 & .5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

30.16. a.

$$\mathbf{b}^* = \begin{bmatrix} -2.077 \\ .724 \end{bmatrix} \quad s = 2.200$$

b.

$t$	$X_1$	$X_2$
1.5	-1.416	.494
2.5	-2.361	.823
3.5	-3.304	1.152